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Qualitative Spatial Reasoning about Cardinal Directions[1]

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Abstract

Spatial reasoning is very important for cartography and GISs. Most known methods translate a spatial problem to an analytical formulation to solve quantitatively. This paper shows a method for formal, qualitative reasoning about cardinal directions. The problem addressed is how to deduce the direction from A to C, given the direction from A to B and B to C. It first analyzes the properties formal cardinal direction system should have. It then constructs an algebra with the direction symbols (e.g., {N, E, S, W}) and a combination operation which connects two directions. Two examples for such algebras are given, one formalizing the well-known triangular concept of directions (here called cone-shaped directions) and a projection-based concept. It is shown that completing the algebra to form a group by introducing an identity element to represent the direction from a point to itself simplifies reasoning and increases power. The results of the deductions for the two systems agree, but the projection bases system produces more 'Euclidean exact' results, in a sense defined in the paper.

1. Introduction

Humans reason in various ways and in various situations about space and spatial properties. The most common examples are navigational tasks in which the problem is to find a route between a given starting point and an end point. Many other examples, such as decisions about the location of a resource, which translates in a mundane household question like "where should the phone be placed?", or the major problem of locating a nuclear waste facility require spatial reasoning. Military applications using spatial reasoning for terrain analysis, route selection in terrain, and so on. (Piazza

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and Pessaro 1990) are frequent. Indeed, spatial reasoning is so widespread and common that it is often not recognized as a special case of reasoning.

Spatial reasoning is a major requirement for a comprehensive GIS and several research efforts are currently addressing this need (Abler 1987, p. 306, NCGIA 1989, p. 125, Try and Benton 1988). It is important that a GIS can carry out spatial tasks, which include specific inferences based on spatial properties, in a manner similar to a human expert and that there are capabilities that explain the conclusions to users in terms they can follow (Try and Benton 1988, p. 10). In current GIS systems, such spatial reasoning tasks are most often formalized by translating the situation to Euclidean geometry then using an analytical treatment for finding a solution. This is admittedly not an appropriate model for human reasoning (Kuipers 1978, p.143) and thus does not lead to acceptable explanations, but Euclidean geometry is a convenient and sometimes the only known model of space available for rigorous analytical approaches. A similar problem was found in physics, where the well known equations from the textbook were not usable to build expert systems. Using more qualitative than quantitative approaches, a formalization of the physical laws we use in our everyday lives was started, the so called 'naive physics' (Hayes 1985, Hobbs and Moore 1985, Weld and de Kleer 1990).

This paper addresses a small subset of spatial reasoning, namely qualitative reasoning with cardinal directions between point-like objects. We assume a 2-dimensional space and exclude radial reference frames, as is customary in Hawaii (Bier 1976). We want to establish rules for inference from a set of directional data about some points to conclude other directional relations between these. We follow McDermott and Davis (1984, p. 107) in assuming that such basic capabilities are necessary for solving the more complex spatial reasoning problems. A previous paper with the terms 'qualitative reasoning' in its title (Dutta 1990) is mostly based on analytical geometry. In contrast, our treatment is entirely qualitative and we use Euclidean geometry only as a source of intuition in Section 4 to determine the desirable properties of reasoning with cardinal directions.

Similarly, the important field of geographic reference frames in natural language (Mark, et al. 1987) has mostly been treated using an analytical geometry approach. Typically, spatial positions are expressed relative to positions of other objects. Examples occur in everyday speech in forms like "the church is west of the restaurant". In the past these descriptions were translated into Cartesian coordinate space and the mathematical formulations analyzed. A special problem is posed by the inherent uncertainties in these descriptions and the translation of uncertainty into an analytical format. McDermott and Davis (1984) introduced a method using 'fuzz' and in (Dutta 1988) and (Dutta 1990) fuzzy logic (Zadeh 1974) is used to combine such approximately metric data.

The problem addressed in this paper, described in practical terms, is the following: In an unknown country, one is informed that the inhabitants use 4 cardinal directions, by the names of 'al' 'bes' 'cel' and 'des', equally spaced around the compass. One also receives information of the type

> Town Alix is al of Beta, Celag is cel of Diton, Beta is des of Diton, Efag is cel of Beta, etc.

We show how one can assert that this is sufficient information to conclude that Alix is al of Efag.

Our concern is different from Peuquet (Peuquet and Zhan 1987), who gave 'an algorithm to determine the directional relationship between arbitrarily-shaped polygons in the plane'. She started with two descriptions of the shape of two objects given in coordinate space and determined the directional relationship (we say the cardinal direction) between the two objects. We are here concerned with several objects. Cardinal directions are given for some pairs of them and we are interested in the rules of inference that can be used to deduce others.

This paper lists a set of fundamental properties cardinal directions should have and defines what exact and approximate qualitative spatial reasoning means. It then gives two possible methods to construct a system of cardinal directions. They seem quite different, one based on a cone shaped or triangular area for a direction, the other based on projections, but they result in very similar conclusions. The projection based is slightly more powerful and easier to describe. The set of desirable properties are formally contradictory and contain some approximate rules, but these seem to pose more of a theoretical than a practical problem; however, clearly more research is necessary to clarify this point.

An approach that is entirely qualitative, and thus similar to the thrust in this paper, is the work on symbolic projections. It translates exact metric information (primarily about objects in pictures) in a qualitative form (Chang, et al. 1990, Chang, et al. 1987). The order in which objects appear, projected vertically and horizontally, is encoded in two strings, and spatial reasoning, especially spatial queries, are executed as fast substring searches (Chang, et al. 1988).

This work is part of a larger effort to understand how we describe and reason about space and spatial situations. Within the research initiative 2, 'Languages of Spatial Relations' of the NCGIA (NCGIA 1989) a need for multiple formal descriptions of spatial reasoning—both quantitative-analytical and qualitative—became evident (Frank 1990, Frank and Mark 1991, Mark and Frank 1990, Mark, et al. 1989). Terence Smith presented some simple examples during the specialist meeting .

> "The direction relation NORTH. From the transitive property of NORTH one can conclude that if A is NORTH of B and B is NORTH of C then A must be NORTH of C as well (Mark, et al. 1989)"

The organization of this paper is as follows: In Section 2 we introduce the concept of qualitative reasoning and relate it to spatial reasoning using analytical geometry; we define 'Euclidean exact' qualitative reasoning based on a homomorphism. In the following section, we list the properties of cardinal directions and in Sections 4 and 5 we discuss two different systems for reasoning with directions and compare them. We conclude the paper with some suggestions for future research.

2. Qualitative approach

2.1. Qualitative reasoning

In this paper, we present a set of qualitative deduction rules for a subset of spatial reasoning, namely reasoning with cardinal directions. In qualitative reasoning a situation is characterized by variables which 'can only take a small, predetermined number of values' (de Kleer and Brown 1985, p. 116) and the inference rules use these values and not numerical quantities approximating them. It is clear that the qualitative approach loses some information, but this may simplify reasoning. We assume that a set of propositions about the relative positions of objects in a plane is given and we have to deduce other spatial relationships (Dutta 1990, p. 351)

> "Given: A set of objects (landmarks) and
> A set of constraints on these objects.
> To find: The induced spatial constraints".

The relations we are interested in are the directions, expressed as symbols representing the cardinal direction.

Without debating whether human reasoning follows the structure of propositional logic, we understand that there is some evidence that human thinking is at least partially symbolic and qualitative (Kosslyn 1980, Lakoff 1987, Pylyshyn 1981). Formal, qualitative spatial reasoning is crucial for the design of flexible methods to represent spatial knowledge in GIS and for constructing usable GIS expert systems (Buisson 1990, McDermott and Davis 1984). Spatial knowledge is currently seldom included in expert systems and is considered 'difficult' (Bobrow, et al. 1986, p.887).

In terms of the example given in the introduction, the following chain of reasoning deduces a direction from Alix to Efag:

1. Use 'Alix is al of Beta' and 'Efag is cel of Beta', two statements which establish a sequence of directions Alix - Beta - Efag.
2. Deduce 'Beta is al of Efag' from 'Efag is cel of Beta'
3. Use a concept of transitivity: 'Alix is al of Beta' and 'Beta is al of Efag' thus conclude 'Alix is al of Efag'.

We shall formalize such rules and make them available for inclusion in an expert system.

2.2. Advantage of qualitative reasoning

A qualitative approach uses less precise data and therefore yields less precise results than a quantitative one. This is highly desirable (Kuipers 1983, NCGIA 1989, p. 126), because

- precision is not always desirable, and
- precise, quantitative data is not always available.

Qualitative reasoning has the advantage that it can deal with imprecise data and need not translate it to a quantitative form. Verbal descriptions are typically not metrically precise, but are sufficient for finding the way to a friend's home, for example. Imprecise descriptions are necessary in query languages where one specifies some property that the requested data should have, for example a building about 3 miles from town. It is difficult to show this in a figure, because the figure is necessarily overly specify or very complex. Qualitative reasoning can also be used for query simplification to transform a query from the form in which it is posed to another, equivalent one that is easier to execute.

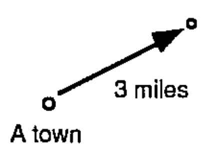


Figure 1: Overspecific visualization

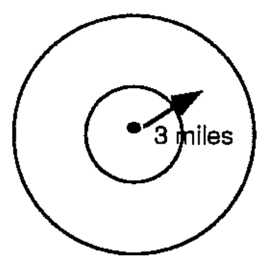


Figure 2: Complex visualization

In other cases, the available data is in qualitative form, most often text documents. For example, (Tobler and Wineberg 1971) tried to reconstruct spatial locations of historic places from scant descriptions in a few documents. Verbal information about locations of places can leave certain aspects imprecise and we should be able to simulate the way humans deduce information from such descriptions, (for example in order to automatically analyze descriptions of locations in natural science collections) (McGranaghan 1988, McGranaghan 1989, McGranaghan 1989).

2.3. Exact and approximate reasoning

We compare the result of a qualitative reasoning rule with the result we obtain by translating the data into analytical geometry and applying the equivalent functions to them. If the results are always the same, i.e., if we have a homomorphism, we call the qualitative rule **Euclidean exact.** If the qualitative rule produces results, at least for some data values, which are different from the ones obtained from analytical geometry, we call it **Euclidean approximate.**

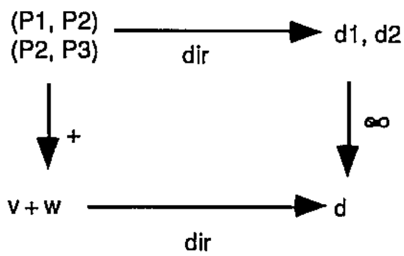


Figure 3: Homomorphism

This is a general definition, which applies to the operation to combine two directions and deduce the direction of the resultant (introduced in 4.3, see figure 5). We establish a mapping from analytical geometry to symbolic directions using a function dir (P1, P2), which maps from a pair of points in Euclidean space to a symbolic direction (e.g., west). Vector addition, with the regular properties is carried to (i.e., replaced with) the symbolic combination ∞.

DEFINITION: a rule for qualitative reasoning on directions is called **Euclidean exact** (for short 'exact') if dir (P_1, P_2) is a homomorphism (Figure 3).

$$\mathrm{dir}(P_1,P_2) \infty \mathrm{dir}\,(P_2, P_3) = \mathrm{dir}\,((P_1, P_2) + (P_2, P_3))$$

2.4. Formalism used

Our method is algebraic (specifically, a relation algebra) and the objects we operate on are the direction symbols S for south, E for west, not the points in the plane. Arguments involving pairs of points, standing for line segments between them, are used only to justify the desirable properties we list.

An algebra consists of

- a set of symbols D, called the domain of the algebra - comparable to the concept of data type in computer programming languages (e.g., D = {N, E, W, S}
- a set of operations over D, comparable to functions in a computer program (primarily operations to reverse and to combine directions), and
- a set of axioms that set forth the basic rules explaining what the operations do (Gill 1976, p. 94).

Specifically, we write (P_1, P_2) for the line segment from P_1 to P_2, and dir $(P_1,P_2) = d_1$ for the operation that determines the direction between two points P_1 and P_2, with d_1 the direction from P_1 to P_2 expressed as one of the cardinal direction symbols.

3. General properties of directions between points

We are interested in two types of operations applicable to direction:

- the reversing of the order of the points and thus the direction of the line segment (the inverse operation), and
- the combination of two directions between two pairs of consecutive points (the combination operation).

Using geometric figures and conclusions from manipulations of line segments, we deduce here properties of these two operations. These properties form then the basis for the qualitative reasoning systems defined in the next two sections.

We define direction as a function between two points in the plane that maps to a symbolic direction:

dir: p x p -> D.

The symbols available for describing the direction depend on the specific system of directions used, e.g., {N, E, S, W} or more extensive {N, NE, E, SE, S, SW, W, NW}.

In the literature, it is often assumed that the two points must not be the same, i.e., the direction from a point to itself is not defined. We introduce a special symbol, which means 'two points too close that a meaningful direction can be determined', and call it the identity element 0. This makes the function total (i.e., it has a result for all values of its arguments),

for all P dir (P, P) = 0.

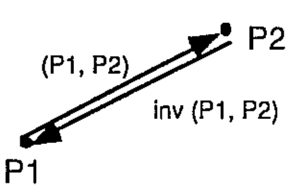
3.1. Reversing direction

Cardinal directions depend on the order in which one travels from one point to the other. If a direction is given for a line segment between points P_1 and P_2, we need to be able to deduce the direction from P_2 to P_1(Figure 4). Already (Peuquet and Zhan 1987) and (Freeman 1975) have stressed the importance of this operation: “Each direction is coupled with a semantic inverse”. We call this 'inverse' (this name will be justified in 4.3.5) written as 'inv' .

inv: d -> d such that inv (dir (P_1, P_2)) = dir (P_2, P_1)

and

inv (inv (d)) = d because inv (inv (P1,P2)) = inv (P_2, P_1) = (P1,P2).



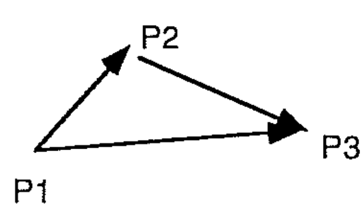


Figure 4: Inverse

Figure 5: Combination

3.2. Combination

Two directions between two contiguous line segments can be combined into a single one. The combination operation is defined such that the end point of the first direction is the start point of the second.

comb : d x d -> d , always written in infix format: $d_1 \infty d_2 = d_3$

with the meaning:

$$\text{dir}\ (P_1,P_2) \infty \text{dir}\ (P_2, P_3) = \text{dir}\ (P_1, P_3).$$

This operation is not commutative, but is associative, and has an identity and an inverse.

Combinations of more than two directions should be independent of the order in which they are combined (**associative** law) and we need not use parenthesis:

$$a \infty (b \infty c) = (a \infty b) \infty c = a \infty b \infty c \text{ (associative law)}$$

This rule follows immediately from Figure 6 or from the definition of combination:

$$\text{dir}\ (P_1, P_2) \infty (\ \text{dir}\ (P_2, P_3) \infty \text{dir}\ (P_3,P_4)\ ) =$$
$$\text{dir}\ (P_1, P_2) \infty \text{dir}\ (P_2, P_4) = \text{dir}\ (P_1, P_4)$$
$$(\ \text{dir}\ (P_1, P_2) \infty \text{dir}\ (P_2, P_3)\ ) \infty \text{dir}\ (P_3,P_4) =$$
$$\text{dir}\ (P_1, P_3) \infty \text{dir}\ (P_3, P_4) = \text{dir}\ (P_1, P_4)$$

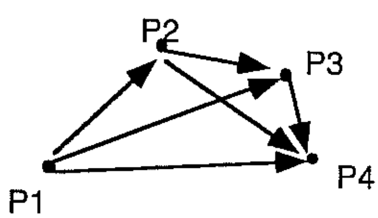


Figure 6: Associativity

The definition of an identity element states that adding the direction from a point to itself, dir (P_1,P_1) to any other direction should not change it.

$$d \infty 0 = 0 \infty d = d \text{ for any d.}$$

In algebra, an inverse to a binary operation is defined such that a value combined with its inverse, results in the identity value. From Figure 4 it follows that this is just the inverses of the given line segment:

$$\text{dir}\ (P_1, P_2) \infty \text{dir}\ (P_2, P_1) = \text{dir}\ (P_1,P_1).$$

In case that two line segments are selected as in Figure 7, such that

$$\text{dir}\ (P_1, P_2) = d_1 \quad \text{and dir}\ (P_2, P_3) = d_2 = \text{inv}\ (d_1)$$

computing the combination

$$\text{dir}\ (P_1, P_2) \infty \text{dir}\ (P_2, P_3) = d_1 \infty \text{inv}\ (d_1) = 0$$

is an approximation and not Euclidean exact. The degree of error depends on the definition of 0 used and the difference in the size of the line segments - if they are the same, the inference rule is exact.

This represents a type of reasoning like New York is east of San Francisco, San Francisco is west of Philadelphia; thus the direction from New York to Philadelphia is 'too close' in this reference frame to determine a direction different from 'the same point' (which is defined here as an additional element of the possible values for a cardinal direction).

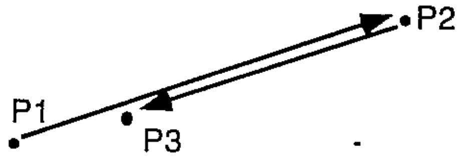


Figure 7: $d \infty \text{inv}(d)$

We find that this combination is 'piece-wise' invertable:

$\text{inv}(a \infty b) = \text{inv}(a) \infty \text{inv}(b)$.

Combinations of directions must have the special property that combining two line segments with the same direction results in the same direction. In a relation-oriented approach, this is a transitivity rule (as quoted in the introduction).

$\text{dir}(P_1, P_2) = \text{dir}(P_2, P_3) = d$ then $\text{dir}(P_1, P_3) = d$
or short: $d \infty d = d$, for any d.

3.3. Summary of Properties of Cardinal Directions

The basic rules for cardinal directions and the operations of inverse and combination are:

- The combination operation is associative (1').
- The direction between a point and itself is a special symbol 0, called *identity* (1) (2')
- The direction between a point and another is the *inverse* of the direction between the other point and the first (2) (3').
- Combining two equal directions results in the same direction (*idempotent*, transitivity for direction relation) (3).
- The combination can be inverted (4).
- Combination is piece-wise invertible (5).

$\text{dir}(P_1, P_1) = 0$	(1)	$d \infty (d \infty d) = (d \infty d) \infty d$	(1')
$\text{dir}(P_1, P_2) = \text{inv}(\text{dir}(P_2, P_1))$	(2)	$d \infty 0 = 0 \infty d = d$	(2')
$d \infty d = d$	(3)	$d \infty \text{inv}(d) = 0$	(3')
for any a, b in D exist unique x in D such that $a \infty x = b$ and $x \infty a = b$	(4)		

inv (a ∞ b) = inv (a) ∞ inv (b) (5)

Properties of direction	Group properties

Several of the properties of directions are similar to properties of algebraic groups or follow immediately from them. Unfortunately, the idempotent property (transitivity for direction relation) (3) is in contradiction with the remaining postulates, especially the definition of identity (3'). Searching for an inverse x for any d ∞ x = 0, we find x = d (using (3)) or x = 0 (using 3'), which contradicts the uniqueness of x (4). It is thus impossible to construct a system which fulfills all requirements at the same time. Human reasoning seems not to insist on associativity.

4. Cardinal directions as cones

The most often used, prototypical concept of cardinal directions is related to the angular direction between the observer's position and a destination point. This direction is rounded to the next established cardinal direction. The compass is usually divided into 4 major cardinal directions, often with subdivisions for a total of 8 or more directions. This results in cone shaped areas for which a symbolic direction is applicable. We limit the investigation here to the case of 4 and 8 directions. This model of cardinal direction has the property that 'the area of acceptance for any given direction increases with distance' (Peuquet and Zhan 1987, p. 66) (with additional references) and is sometimes called 'triangular'.

4.1. Definitions with 4 directional symbols

We define 4 cardinal directions as cones, such that for every line segment, exactly one direction from the set of North, East, South or West applies.

for every P_1, P_2 ($P_1 \neq P_2$) exist d (P_1, P_2) with d in D_4 ={N, S, E, W}.

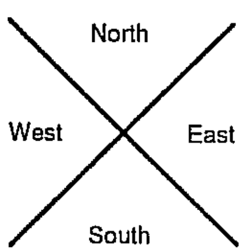


Figure 8: Cone-shaped directions

An obvious operation on these directions is a quarter-turn, anti-clock-wise (mathematically positive) q, such that

q: d -> d, with q(N) = E, q(E) = S, q(S) = W, q (W) = N

and four quarter turns are an identity:

$q (q (q (q (d)))) = q^4 (d) = d.$

Reversing a direction is equal to 2 quarter turns (or one half turn)

$inv (d) = q^2 (d).$

Finally, we just define the combination of two directions, such that transitivity holds

$d \infty d = d$

but every other combination remains undefined.

These definitions would fulfill the requirements for the direction except that we did not define a symbol for identity. Very few combinations of symbols produce results.

4.2. Completion with identity

Introducing an identity element, we eliminate the restriction in the input values for the direction function

for every P_1, P_2 exist d (P_1, P_2) with d in $D_5 = \{N, S, E, W, 0\}$.

A quarter turn on the identity element 0 is 0

$q (0) = 0$

and thus

$inv (0) = 0$ from $q(q(0)) = q (0) = 0$

$d \infty 0 = 0 \infty d = d$ from group properties

$0 \infty 0 = 0$ from $d \infty d = d$.

The inverse must further have the property that a direction combined with its inverse is 0

$d \infty inv (d) = 0.$

These definitions contain the previously listed ones as subset D_4 (not subgroup, because identity is not in the subset). Both the set D_5 and the subset D_4 is closed under the operations 'inverse' and 'combination'.

From the total of 25 different combinations, one can only infer 13 cases exact and 4 approximate; other combinations do not yield an inference result with these rules. Summarized in a table (lower case indicate approximate reasoning):

	N	E	S	W	0
N	N		o		N
E		E		o	E
S	o		S		S
W		o		W	W
0	N	E	S	W	0

4.3. Directions in 8 or more cones

One may use a set of 8 cardinal directions D_9 = {N, NE, E, SE, S, SW, W, NW, 0}, using exactly the same formulae. In lieu of a quarter turn, we define a turn of an eighth:

e (N) = NE, e (NE) = E, e (E) = SE, ... , e (NW) = N, e (0) = 0

with 8 eighth turns being the identity

e^8 (d) = d

and inverse now equal to 4 eighth turns

inv (d) = e^4 (d).

All the rules about combination of direction, etc., remain the same and one can also form a subset {N, NE, E, SE, S, SW, W, NW} without 0.

An approximate averaging rule combines two directions that are each one eighth off. For example, SW combined with SE should result in S, or N combined with E should result in NE.

e (d) ∞ -e (d) = d

with -e (d) = e^7 (d), or one eight turn in the other direction)

One could also assume that if two directions are combined that are just one eights turn apart, one selects one of the two (S combined with SE results in S, N combined with NW results in NW).

e(d) ∞ d = d and d ∞ e(d) = d

Human beings would probably round to the simple directions N, E, W, S, but formalizing is easier if preference is given to the direction which is second in the turning direction. This is another rule of approximate reasoning.

This rule can then be combined with other rules, for example to yield (approximate)

e(d) ∞ inv d = 0 and e(d) ∞ e (inv (d)) = 0.

In this system, from all the 81 pairs of values (64 for the subset without 0) combinations can be inferred, but most of them only approximately. Only 24 cases (8 for the subset) can be inferred exactly; 25 result in a value of 0 and another 32 give approximate results. We can write it as a table, where lower case denotes Euclidean approximate inferences:

	N	NE	E	SE	S	SW	W	NW	0
N	N	n	ne	o	o	0	nw	n	N
NE	n	NE	ne	e	o	o	o	*N*	NE
E	ne	ne	E	e	se	o	o	o	E
SE	o	e	e	SE	se	s	o	o	SE
S	o	o	se	se	S	s	sw	o	S
SW	o	o	o	s	s	SW	sw	w	SW
W	nw	o	o	o	sw	sw	W	w	W
NW	n	n	o	o	o	w	w	NW	NW
0	N	NE	E	SE	S	SW	W	NW	0

5. Cardinal directions defined by projections

5.1. Directions in 4 half-planes

Four directions can be defined, such that they are pair-wise opposites and each pair divides the plane into two half-plains. The direction operation assigns for each pair of points a combination of two directions, e.g., South and East, for a total of 4 different directions. This is an alternative semantic for the cardinal direction, which can be related to Jackendoff's principles of centrality, necessity and typicality (Jackendoff 1983, p. 121). Peuquet pointed out that directions defined by half-planes are related to the necessary conditions, whereas the cone-shaped directions give the typical condition (Mark, et al. 1989, p. 24).

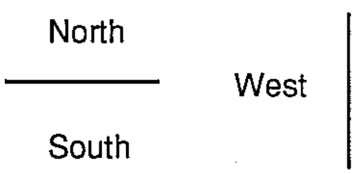


Figure 9: Two sets of half-planes

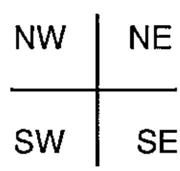


Figure 10: Directions defined by half-planes

Another justification for this type of reasoning is found in the structure geographic longitude and latitude imposes on the globe. Cone directions better represent the direction of 'going toward', whereas the 'half-plane' (or equivalent parts of the globe) better represents the relative position of points on the earth. However, the two coincide most of the time. To reach an object which is $\text{north}_{\textit{half-plane}}$ on the globe one has to go $\text{north}_{\textit{cone}}$.

For half-plane directions, one defines the cardinal directions as different from each other and E - W and N - S pair-wise inverse (Peuquet and Zhan 1987, p. 66). In this system, the two projections can be dealt with individually. Each of them has the exact same structure and we describe first one case separately and then show how it combines with the other.

The N-S case, considered the prototype for the two cases E-W and N-S has the following axioms:

for every P_1, P_2 ($P_1 \neq P_2$) dir_{ns} (P_1, P_2)= d_{ns} with d_{ns} in {N,S}

The inverse operation is defined such that inv (inv (d)) = d holds:

inv (N) = S, inv (S)= N.

Next we define the combination of two directions, such that transitivity holds:

for all d in {N,S} $d \infty d = d$ (which is $N \infty N = N$, $S \infty S = S$)

We now combine the two projections in N-S and E-W to form a single system, in which we have for each line segment one of 4 combinations of directions assigned.

D_4 = { NE, NW, SE, SW}

We label the projection operations by the directions they include (not the direction of the projection):

p_{ns}: d_4 -> d_{ns}, d_{ns} in {N, S}
p_{ew}: d_4-> d_{ew}, d_{ew} in {E, W}

and a composition operation

c: dns x dew -> d_9 such that c (p_{ns} (d), p_{ew} (d)) = d.

The rules for d_{ew} are the same as for d_{ns} explained above, replacing N by E and S by W:

inv (E) = W, inv (W) = E
$E \infty E = E$, $W \infty W = W$.

The inverse operation is defined as the inverse applied to each projection:

inv (d) = c (inv (d_{ns}), inv (d_{ew}))

and combination is similarly defined as combination of each projection

$d_1 \infty d_2 = c\,(d_{ns}(d_1) \infty d_{ns}(d_2),\ d_{ew}(d_1) \infty d_{ew}(d_2))$.

Unfortunately, combination is defined only for the four cases

NE ∞ NE = NE NW ∞ NW = NW
SE ∞ SE = SE SW ∞ SW = SW

and others, like

NE ∞ NW

which should approximately result in N, cannot be computed. This system, lacking an identity, is not very powerful, as only 4 of the 16 combinations can be inferred.

5.2. Directions with neutral zone

We can define the directions such that points which are near to due north (or west, east, south) are not assigned a second direction, i.e., one does not decide if such a point is more east or west. This results in a division of the plane into 9 regions, a central neutral area, four regions where only one direction letter applies and 4 regions where two are used.We define for N-S three values for direction d_{ns} {N, P, S} and for the E - W direction the values d_{ew} {E, Q, W}.

NW	N	NE
W	U	E
SW	S	SE

Figure 11: Directions with neutral zone

It is important to note, that there is no determination of the width of the 'neutral zone' made. Its size is effectively decided when the directional values are assigned and a decision is made that P_2 is north (not north-west or north-east) of P_1. We only assume that these decisions are consistently made. Similar arguments apply to the neutral zone of cone shaped directions, but they are not as important.

Allowing a neutral zone, either for the cone or projection based directions introduces an aspect of 'tolerance geometry'. Strictly, whenever we assign identity direction dir $(P_1, P_2) = 0$ for cases where $P_1 \neq P_2$ we violate the transitivity assumption of equality.

dir $(P_1, P_2) = 0$ and dir $(P_1, P_3) = 0$ need not imply dir $(P_2, P_3) = 0$

A tolerance space (Zeeman 1962) is mathematically defined as a set (in this case the points P) and a tolerance relation. The tolerance relation relates objects which are close, i.e., tol (A, B) can be read A is sufficiently close to B that we can or need not differentiate between them. A tolerance relation is similar to an equality, except that it admits small differences. It is reflexive and symmetric, but not transitive (as an equality would be)

tol (A, B)
tol (A, B) = tol (B, A).

A tolerance relation can be applied to geometric problems (Robert 1973).

Using the same methods as in 5.1 for the definition of the operations in each projection first and then combine them, we find for the inverse operation the following table:

d=	NE	N	NW	E	W	0	SE	S	SW
inv(d)=	SW	S	SE	W	E	0	NW	N	NE

The combination operation, again defined as the combination of each projection, allows one to compute values for each combination. Written as a table (again, lower case indicates approximate reasoning):

	N	NE	E	SE	S	SW	W	NW	0
N	N	NE	NE	e	o	w	NW	NW	N
NE	NE	NE	NE	e	e	o	n	n	NE
E	NE	NE	E	SE	SE	s	o	n	E
SE	e	e	SE	SE	SE	s	s	o	SE
S	o	e	SE	SE	S	SW	SW	w	S
SW	w	o	s	s	SW	SW	SW	w	SW
W	NW	n	o	s	SW	SW	W	NW	W
NW	NW	n	n	o	w	w	NW	NW	NW
0	N	NE	E	SE	S	SW	W	NW	0

The system is not associative, as

$$(N \infty N) \infty S = N \infty S = 0 \text{ but } N \infty (N \infty S) = N \infty N = N.$$

In the half-plane based system of directions with a neutral zone, we can deduce a value for all input values for the combination operation (81 total), 56 cases are exact reasoning, not resulting in 0, 9 cases yield a value of 0, and another 16 cases are approximate.

6. Assessment

The power of the two systems which lack an identity element, the 4 direction cone-shaped and the 4 half-plane directional system, is very limited; most combinations cannot be resolved. The two systems with 8 direction and identity, the 8 direction cone-shaped and the 4 projection based directional system, are comparable. Each system uses 9 directional symbols, 8 cone directions plus identity on one hand, the Cartesian product of 3 values (2 directional symbols and 1 identity symbol) for each projection on the other hand. The reasoning process in the half-plane based system uses fewer rules, as each projection is handled separately with only two rules. The cone-shaped system uses two additional approximate rules which are then combined with the other ones. An actual implementation would probably use a table look-up for all combinations and this would not make a difference.

Both systems violate some of the desired properties. One can easily observe that associativity is not guaranteed, but the differences seem to not be very significant.

An implementation of these rules and comparison of the computed combinations with the exact value was done and confirms the theoretical results. Comparing all possible 10^6 combinations in a grid of 10 by 10 points (with a neutral zone of 3 for the projection based directions) shows that the results for the projection based directions are correct in 50% of the cases and in only 25% for cone-shaped directions. The result 0 is the outcome of 18% of all cases for the projection based, but 61% for the cone-shaped directions. The direction-based system with an extended neutral zone produces a result in 2% of all cases that is a quarter turn off, otherwise the deviation from the correct result is never more than one eighth of a turn (namely in 13% of all cases for cone-shaped and 26% for projection based direction systems). In summary, the projection based system of directions produces a result in 80% of all cases that is within 45° and otherwise the value 0.

7. Conclusions

This paper introduces a system for inference rules for completely symbolic, qualitative spatial reasoning with cardinal distances. We have first stressed the need for symbolic, qualitative reasoning for spatial problems. It is important to construct inference systems which do not rely on quantitative methods and need not translate the problem to analytical geometry, as most of the past work did. The systems investigated are capable of resolving any combination of directional inference using a few rules. Returning to our example in the introduction, we cannot only assert that Alix is al of Efag, but also that Alix is al-des from Diton and Celag, etc.

We used geometric intuition and the definition of a direction as linking two points. From this we deduced a number of desirable properties for a system to deal with cardinal directions. We use an algebraic approach and define two operations, namely inverse and combination. We found several properties, e.g.,

- the direction from a point to itself is a special value, meaning 'too close to determine a direction'
- every direction has an inverse, namely the direction from the end point to the start point of the line segment
- the combination of two line segments with the same direction result in a line segment with the same direction.

We defined the notion of 'Euclidean exact' and 'Euclidean approximate' as properties of a qualitative spatial reasoning system. A deduction rule is called 'Euclidean exact' if it produces the same results as Euclidean geometry operations would.

We then investigated two system for cardinal directions, both fulfilling the requirements for directions. One is based on cone-shaped (or triangular) directions, the other deals with directions in two orthogonal projections. Both systems, if dealing with 4 cardinal directions, are very limited and when dealing with 8 directions, still weak. The introduction of the identity element simplifies the reasoning rules in both cases and increases the power for both cone and projection based directional systems. The deductions in this section use only the algebraic properties and does not rely on geometric intuition or properties of line segments.

Both systems yield results for all the 81 different inputs for the combination operation. But the projection based system more often yields an Euclidean exact result than the cone based one (49 vs. 25 cases). It also produces the value 0 less often (9 vs. 25 cases).

Another important result is that the two systems do not differ substantially in their conclusions, if definite conclusions can be drawn, i.e., not the value 0. This reduces the potential for testing with human subjects to find out which system they use, observing cases where the conclusion to use one or the other line of reasoning would yield different results.

We have implemented these deduction rules and compared the results obtained for all combinations in a regular grid. The projection based system results in 53% of all cases in exact results and in another 26% in results which are not more than 45° off. In 18% of all cases the application of the rules yields a value of 0. The results for the cone-shaped directions are less accurate. It will be interesting to see how this accuracy compares with human performance but also if it is sufficient for expert systems and for query and search optimization. The methods shown here can be used to quickly assess if the combination of two directions yields a value that falls within some limits and thus a more accurate and slower computation should be done.

There is not much previous work on qualitative spatial reasoning and several different directions for work remain open:

- Qualitative reasoning using distances,
- Combining reasoning with distances and directions,
- Hierarchical system for qualitative reasoning,
- Directions of extended objects, and
- Reasoning systems, human beings use.

Qualitative reasoning using distances - There is a good, mathematically based definition for distance measures expressed as real numbers. This can probably be carried over to qualitative distance expression, e.g., {Near, Far} or {Near, Intermediate and Far}, and rules for symbolic combinations similar to the one listed here deduced.

Combining reasoning with distances and directions - Combining the reasoning with directions and distances can be more than just combining two orthogonal systems; there are certainly interesting interactions between them (Hernández 1990). Most of the approximate reasoning rules are based on the assumption that the distances between the points discussed are about equal. This is not as unreasonable as it may sound, as directional reasoning is probably more often carried out regarding objects of the same import and thus at about the same distance. Nevertheless, it is a weak assumption and further work should approach spatial reasoning on distances and then combine the two.

Hierarchical systems for qualitative reasoning - A system for reasoning with distances differentiating only two or three steps of farness is quite limited. Depending on the circumstances a distance appears far or near compared to others. One could thus construct a system of hierarchically nested neighborhoods, wherein all points are about equally spaced. Such a system can be formalized and may quite adequately explain some forms of human spatial reasoning.

Distances and directions of extended objects - The discussion in this paper dealt exclusively with point-like objects. This is a severe limitation and avoided the difficult problem of explaining distances between extended objects. Peuquet in (Peuquet and Zhan 1987) tried to find an algorithm that gives the same result than 'visual inspection'; however, visual inspection does not yield consistent results. It might be useful to see if sound rules, like the above developed ones, may be used to resolve some of the ambiguities.

What system of qualitative reasoning do humans use? - We can also ask, which one of the systems proposed humans use. For this, one has to see in which cases different systems produce different results and then test human subjects to see which one they employ. This may be difficult for the cone and projection based direction system, as their deduction results are very similar. Care must be applied to control for the area of application, as we suspect that different types of problems suggest different types of spatial reasoning.

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