Experiential and Formal Models of Geographic Space[1]

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Abstract

This paper is concerned not with space and spatial relations as objective entities of the world, but rather with human experience and perception of phenomena and relations in space. The goal arising from this concern is to identify models of space that can be used both in cognitive science and in the design and implementation of geographic information systems (GISs). Experiential models of the world are based on sensorimotor and visual experiences with environments, and form in individual minds as the associated bodies and senses experience their worlds. Formal models consist of axioms expressed in a formal language, together with mathematical rules to infer conclusions from them. The paper reviews both kinds of models, viewing them each as abstractions of the same 'real world.' The review of experiential models is grounded in recent developments in cognitive science, expounded by Rosch, Johnson, Talmy, and especially Lakoff. Among other things, these models suggest that perception and cognition are driven by schemata and other mental models, often language-based. These models form a framework for a review of models of small-scale spaces filled with everyday objects. The ways in which people interact with such spaces is in sharp contrast to the bit-by-bit experience with geographic (large-scale) spaces during wayfinding and other spatial activities. The paper then addresses the issue of the 'objective' geometry of geographic space. If objectivity is defined by measurement, this leads to a surveyors' view, and a near-Euclidean geometry. The paper then relates these models to issues in the design of GISs. To be implemented on digital computers, geometric concepts and models must be formalized. The idea of a formal geometry of natural

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language is discussed, and some aspects of it are presented. Formalizing the link between cognitive categories and models on the one hand, and geometry and computer representations on the other is a key element in the research agenda.

1. Introduction

Spatial relations do not *exist* in the real world; rather, they exist in minds, to aid in making sense of the world, and in interacting with it. Our concern therefore is not with space and spatial relations as objective entities of the world, but rather is with human experience and perception of phenomena and relations in space. This is a strong statement, and one which appears to be at odds with the positivist paradigm. However, in this paper, we present an approach from cognitive science, and apply it to geographic space and spatial relations. This approach attempts to avoid some of the fundamental inconsistencies that are embedded within the positivist approach and the scientific method, yet avoids falling into pure solipsism[3], losing the ability to discuss the relevant observations and to propose formal models. More specifically, this paper discusses and compares two kinds of models that can be used to define space and spatial relations: experiential models and formal models.

Experiential models of the world are based on sensorimotor and visual experiences with our environments. The experiential models form in individual minds as the associated bodies and senses experience their worlds. Due to the physiological similarities that exist among individual human beings, it appears that most people experience their environments in similar ways. Thus, we can expect that the basic features of individual experiential models of geographic space, while inherently personal, will have much in common across individuals. Experiential models of space can reveal themselves through spatial reference in natural language, through experiments with human subjects, through observation of spatial behavior, or through study of the artifacts of such behavior. *Experiential realism*, a philosophical basis for cognitive science that has recently been advanced by George Lakoff (1987) and Mark Johnson (1987), and discussed in a geographic context by Couclelis (1988), Mark and Frank (1989), and Mark (1989), is central to the models discussed here.

Formal models consist of axioms expressed in a formal language, together with mathematical rules to infer conclusions from them. We will review the use of such models as they are used to represent geographic space and

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[3] Solipsism is a philosophical position which denies the existence of the "real world", or at least insists that human minds can have no direct access to such a world, but knows only what the senses 'tell it',

spatial relations. We will typically use results from geometry, topology and algebra in our quest to build formal models that are useful for geography. Formal models often bear strong similarities with experiential models of space and spatial objects; this is because both experiential and formal models often have been developed as *abstractions of the same aspects of human observation and experience with the same world*. For example, as we will discuss below, Euclidean geometry is said to have begun through the formalization (by Euclid and others) of rules and procedures used for land surveying in ancient Egypt. This geometry is fully consistent with Newtonian (solid-body) physics; however, Newtonian physics itself corresponds closely with naive (experiential) physics in many every-day situations.

Our approach differs from most previous work on geographic theory, in that it draws on concepts related to human natural language. Most previous work on spatial cognition in geography has concentrated on studies of human behavior (for examples, see Golledge and Zannaras, 1973; Golledge, 1976, 1978; Golledge *et al.*, 1983; Golledge, 1988). In a departure, the recent *Annals* paper by Peuquet (1988), emphasized results from studies of human vision. In contrast, the conceptual basis of our work is found primarily in the more linguistic parts of cognitive science. Our approach draws heavily on the work of Eleanor Rosch (Rosch, 1973, 1978), Leonard Talmy (Talmy, 1983), George Lakoff (Lakoff and Johnson, 1980; Lakoff, 1987), Annette Herskovits (Herskovits, 1982, 1985, 1986), Mark Johnson (Johnson, 1987), and others in Cognitive Science. Although a few articles drawing on this literature have already been published in the geographic and GIS literatures (Mark, Svorou, and Zubin, 1987; Couclelis, 1988; Mark, 1989; Mark and Frank, 1989; Mark, Gould, and Nunes, 1989), this paper extends this work substantially.

2. GIS and Theoretical Geography

Geographers have long sought to develop a theory or theories of geographic space, or perhaps geographic theories of space in general. Recent developments in geographic information systems (GIS) have brought out renewed calls for 'general' theories of spatial relations (Boyle *et al*., 1983; Abler, 1987; Frank, 1987; Peuquet, 1988; NCGIA, 1989). Although theories of space and spatial relations need not have the explanatory power of the theories of a prototypical 'science' such as physics, GISs cannot be built without them. Furthermore, in a formal sense, a computer program can be considered to be a statement of some theory, and in this sense any GIS already is, or at least contains, geographic theory. If a more rigorous and explanatory definition of 'theory' is used, GIS certainly can be a test-bed for evaluating geographic theory.

Thus, it is not surprising that one of the five high-priority topics for research by the National Center for Geographic Information and Analysis (NCGIA) is "a general theory of spatial relationships" (Abler, 1987, p. 304). Abler goes on to elaborate that the goal is "a coherent, mathematical theory of spatial relationships" (Abler, 1987, p. 306). On the same page, he also states:

> "Fundamental spatial concepts have not been formalized mathematically and elegantly. Cardinal directions are
> relative concepts, as are ideas basic to geography such as near, far, touching, adjacent, left of, right of, inside, outside, above, below, upon, and beneath." But it is not sufficient for a "theory of spatial relationships" to be mathematically elegant. The concepts embedded in such a theory also must correspond with the concepts used by human minds as parts of spatial cognition, spatial reasoning, and spatial behavior; otherwise, it will be of little if any use to geographers, spatial analysts, or geographic information systems (GIS) users. Thus the search for "fundamental spatial concepts" must be conducted in the cognitive sciences in parallel with searches in mathematics (NCGIA, 1989).

Of course, this search for fundamental spatial concepts is not new. Blaut's (1961) *Space and Process*, Bunge's (1962) *Theoretical Geography*, and Sack's (1973) *Geography, Geometry, and Explanation* represent three of the more prominent of such efforts. Geographical theory has often appeared to be mathematical, and has sometimes been connected to language. For example, geometry was discussed by Harvey (1969, pp. 191-229) as "the language of spatial form." And, more than a decade ago, several papers at the Harvard symposium on data structures for GIS addressed just these issues, and provided a number of approaches (in particular, see Chrisman, 1979; Kuipers, 1979; Sinton, 1979; Youngman, 1979).

The need for theory in GIS was even more clearly expressed in 1983, when, in the report of a NASA-sponsored meeting, it was recognized that:

> The (present) lack of a coherent theory of spatial relations hinders the use of automated geographic information systems at nearly every point. It is difficult to design efficient databases, difficult to

> phrase queries of such databases in an effective way, difficult to interconnect the various subsystems in ways which enhance overall system function, and difficult to design data processing algorithms which are effective and efficient. As we begin (to) work with very large or global spatial databases the inabilities and inefficiencies which result from this lack of theory are likely to grow geometrically.

While we can continue to make some improvement in the use of automated geographic information systems without such a coherent theory on which to base our progress, it will mean that the development will rest on an inevitably shaky base and that progress is likely to be much slower than it might be if we had a theory to direct our steps. It may be that some advances will simply be impossible in the absence of a guiding theory (Boyle *et al*., 1983).

The needs for a sound conceptual basis for GIS, and for a mathematical basis for theories of geographic space, can lead to parallel and complementary research efforts within the GIS agenda, in cognitive science, and in geography in general. Some signposts along this path are presented in this paper.

3. Cognitive Categories and Experiential Realism

The concepts and principles presented in this paper are based on a model of human perception and cognition initiated by Rosch (1973, 1978) and her colleagues, and recently elaborated upon by Lakoff (1987) and particularly by Johnson (1987). The model departs from the classical or set-theoretic view of categories in a number of fundamental ways, and requires some exposition here.

The classical view of categories is that they correspond mathematically to sets (Lakoff, 1987). In fact, it probably is more correct to say that the *mathematical* concept of a set is a formalized version of the *naive* concept of a category. Among the fundamental principles of this set-theoretic model of categories is that there are some necessary and sufficient observable properties of an object, from which its membership in some set can be unambiguously deduced. Another principle is that all members of the set are equally related to the set, and thus would be equally good examples of the set; this classical model thus would predict that, when asked to give an example of a member of a set, a person would be equally likely to name any member of that set as any other.

Experiments in cognitive science find that neither of these aspects is true of the categories that individuals use to characterize their worlds (see Smith and Medin, 1981; also Lakoff, 1987, pp. 54-570). Problems with this classical theory were noted quite early by Cassirer (1923), but the work of Rosch (1973, 1978) was central to the diffusion of doubt about the classical theory. Rosch and her co-workers discovered that all members of a category are not 'equal'. For example, when asked to give an example of a bird, subjects tend to name robins and sparrows far more often than they mention turkeys or penguins. Rosch's data are far more consistent with a model in which a category has a prototype or exemplar (or a small set of these), plus some rules for extending the category, by analogy, metaphor, and other procedures, to more peripheral members. Lakoff (1987) later discussed this in terms of a radial structure for some categories; peripheral members of different arms of a radially-organized class may have nothing in common, except resemblance (in different senses) to a common prototype.

Smith and Medin (1981) reviewed the classical theory, and the problems with it, and proposed two alternative models of categories. One is a probabilistic model; this, however, fails to predict some areas in which observed category structures depart from the classical model. Another model they discuss, at much less length, is one based on exemplars. That model would represent a class by a collection of one or more actual cases which in some sense exemplify the class. Although such a model is highly consistent with observed cognitive data, complete description of all properties of class exemplars seems unlikely, and comparison of a new case with all the exemplars, which would be needed to assign that object to some category, may not be a practical model of the mind. The model proposed by Lakoff (1987) and Johnson (1987) is similar to Smith and Medin's (1981) exemplar model, but is based on idealized prototypes rather than actual-case exemplars.

4. Perception, Cognition, and Schemata

Recent developments in cognitive science suggest that the categories that people use are not necessarily "objective". According to this view, perception and cognition do not involve "direct" interaction with the world, but rather occur through cognitive models, image-schemata, etc. Neisser (1976) discussed how even apparently-direct visual experiences are influenced (biased) by what we expect to see, or what we look for:

> In my view, the cognitive structures crucial for vision are the anticipatory schema that prepare the perceiver to accept certain kinds of information rather than others and thus control the activity of looking. Because we can see only what we know how to look

> for, it is these schema (together with the information actually available) that determine what will be perceived. (Neisser, 1976, p. 20)

Neisser presented the following definition of a schema:

> A schema is that portion of the entire perceptual cycle which is internal to the perceiver, modifiable by experience, and somehow specific to what is being perceived. The schema accepts information as it becomes available at sensory surfaces and is changed by that information; it directs movements and exploratory activities that make more information available, by which it is further modified. (Neisser, 1976, p. 54)

Schemata form a central part of Neisser's model of cognition. Objects are conceptualized through "object schema" (see Neisser, pp. 67-70). He also emphasized the role of schema in wayfinding and navigation:

> I will ... frequently use the term "orienting schema" as a synonym for "cognitive map" to emphasize that it is an active, information-seeking structure. Instead of defining a cognitive map as a kind of image, I will propose ... that spatial imagery itself is just such an aspect of the functioning of orienting schemata. (Neisser, 1976, p. 111).

This theme will be picked up in a later section of this paper. Johnson (1987) claims that mental activities such as perception and cognition are heavily influenced by what he calls *image-schemata*. Johnson defined a schema in a way which is different from, yet consistent with, the definition provided by Neisser:

> "A schema consists of a small number of parts and relations, by virtue of which it can structure indefinitely many perceptions, images, and events. In sum, image-schemata operate at a level of mental organization that falls between abstract propositional structure, on the one side, and particular concrete images on the other." (Johnson, 1987, p. 29)

For any particular domain of investigation, one conceptual schema may be more useful than others. It is more likely that the most appropriate schema will change from problem to problem. Also, the schema themselves may change with each use. It is not an issue of whether one particular schema is "correct" or not, but rather is an issue of how useful some particular schema is for some particular situation.

Johnson (1987, p. 126) provides a clear statement, with examples, of how an image-schemata-based model of cognition would operate:

> "... Much of the structure, value, and purposiveness we take for granted as built into our world consists chiefly of interwoven and superimposed schemata... *My chief point has been to show that these image schemata are pervasive, well-defined, and full of sufficient internal structure to constrain our understanding and reasoning.* [Johnson’s italics] To give some idea of the extent of the image-schematic structuring of our understanding (as our mode of being-in-the-world or our way of having-a world), consider the following partial list of schemata, which includes those previously discussed:
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> Container
> Blockage
> Removal
> Mass-Count
> Center-Periphery
> Scale
> Splitting
> Superimposition
> Process
> Collection
> Balance
> Counterforce
> Enablement
> Path
> Cycle
> Part-whole
> Full-empty
> Iteration
> Surface
> Compulsion
> Restraint
> Attraction
> Link
> Near-Far
> Merging
> Matching
> Contact
> Object
>
> This brief list is highly selective, but it includes what I take to be most of the important image-schemata. If one understands ’schema’ more loosely than I do, it might be possible to extend this list at length." (Johnson, 1987, p. 126).

Note that many of the image-schemata that Johnson lists are inherently spatial or even geographical: CONTAINER, BLOCKAGE, PATH, SURFACE, LINK, NEAR-FAR, CONTACT, CENTER-PERIPHERY, SCALE. Others have implications for spatial language and concepts, spatial interaction modelling, *etc*. (for example, PART-WHOLE and ATTRACTION). For example, Johnson recognizes the importance of ’near’ in his discussion of how image schemata, and in particular the center-periphery schema, constrain meaning, understanding, and rationality:

> "Given a center and a periphery we will experience the NEAR-FAR schema as stretching out along our perceptual or conceptual perspective. What is considered near will depend upon the context, but, once that is established, a SCALE is defined for determining relative nearness to the center." (Johnson, 1987, p. 125)

Lakoff and Johnson point out that in fact, spatial schemata are at the core of cognitive structure, and form the basis for organizing many less-concrete domains. "Spatialization metaphors are rooted in physical and cultural experiences" (Lakoff and Johnson, 1980, p. 18). For example, a physical journey through geographic space becomes a metaphor for various kinds of work projects, and even for interpersonal relationships ("We're at a *crossroads*"; "This relationship is a *dead-end street*"; etc.; Lakoff and Johnson, 1980, p. 44-45). One should note here that this method of 'spatialization' of inherently non-spatial concepts makes results from geography, as the science investigating space and spatial relations, applicable to other domains.

4.1 Some Geographical Examples

Image-schemata are, in principle, not directly observable. However, if Lakoff, Johnson, and the others are correct, image-schemata have a profound and pervasive influence on cognition thought, and language. In this section we will use some examples of natural-language expressions describing geographic situations which allow us to deduce which image schema was likely to have been dominant in the speakers mind at the time the expression was uttered.

Most Indo-European languages express fundamental spatial relations through prepositions. (Some other languages used 'post-positions', cases, or other grammatical structures.) One seemingly-unusual fact about English is that the relations of features (figures) to areal or polygonal reference (ground) regions is expressed by the preposition "in" in some cases yet by "on" in other cases. For example, note the use of "in" and "on" in the following: "I was standing **in** my back yard **on** my property **in** Amherst." Each ground object ("back yard", "property", "Amherst") has a surface, and each has a boundary; thus both "in" and "on" would seem to be valid in each case. Nevertheless, most ground objects do not give the English speaker a choice, but rather require one preposition or the other. Herskovits (1986, p. 147; p. 153) catalogued some cases, but did not provide an explanation. Furthermore, the distinction between ground objects which require "in" and those which require "on" probably is quite old, since, although there are a few exceptions, German and Dutch commonly require **auf** or **op** (respectively) for the same situations for which English uses "on". And, both German and Dutch use **in** for situations in which English also uses "in". Grimaud (1988) has discussed these cases for both English and French.

Mark (1989) provided an explanation for this, which actually changes the question, rather than answers it. Mark (1989) proposed that the choice of preposition *depends on the image schema adopted.* In some cases, a PLATFORM schema is adopted; once this schema is activated, the English preposition "on" is obligatory. (We follow Mark, 1989, in using a new PLATFORM schema rather than Johnson's SURFACE schema, to allow us to use distinct schemata for the German **auf** and **an**, whose distinction will be discussed below.) In other cases, a CONTAINER schema is invoked, forcing the speaker or writer of English to use "in". The question relating to the use of "in" or "on" then becomes: "Which image-schemata are activated for which kinds of ground objects and used in which circumstances?" Finding an answer to this question is a challenging research problem.

Mark (1989) noted that conceptualizing something as an *island* more-or-less forces an English speaker to select the PLATFORM image-schema, and use the preposition "on". If the word "island" appears in the name, this almost requires the speaker to say "on". ("Who lived *on* Manhattan Island before the Europeans came?") On the other hand, for political units, English almost invariably invokes the CONTAINER schema and uses "in". This will be true even for regions that happen to be in 1:1 correspondence with a physical island. ("Does your uncle still live in Puerto Rico?") However, for such island units, either "in" or "on" might be used, and the preposition chosen can indicate whether one is talking about a physical island or a country by forcing the listener/reader to use a particular schema. "Did anyone live on Cuba before 1492?"--the same sentence with "in" might sound strange, since Cuba-the-country did not exist then. (Unlike islands, continents typically take the preposition "in" in English; the relation of choice of schemas to sheer size of the landmass is an open question.)

The following example of how the choice of preposition may force the reader or listener to make different interpretations, based on different image-schemata, was first presented by Mark (1989):

"Hawaii" is the name of a State of the USA; but, "Hawaii" is also the name of the largest and easternmost *island* in that State. Recall that in English, political units normally involve the CONTAINER image-schema, whereas islands use the PLATFORM image-schema. Thus, if I say: "My friend Sherry lives **in** Hawaii", it seems that "in" forces the CONTAINER image-schema, leading to the "State-of-Hawaii" interpretation. She might live in Honolulu (on the Island of Oahu), or anywhere else in the State. But, if I say: "My friend Sherry lives **on** Hawaii", then the PLATFORM image-schema leads to the "Island-of-Hawaii" interpretation, and the residence probably is Hilo or Kona. The use of "in" or "on" forces either the CONTAINER or PLATFORM schema, respectively, thus reducing ambiguity. (Mark, 1989, p. 554)

Natural languages differ in their potential to influence meaning in this way. For example, in Spanish, most locative expressions use more generic prepositions such as **en** (in, on, or at) or **de** (also used as a possessive). Indeed, a dictionary gives the primary meaning of **en** as "*prep.* of time or place" (Velázquez, 1973, p. 267). Thus a Spanish-speaking person would not normally use a choice of prepositions to distinguish the two Hawaiian situations discussed in the last paragraph, but would have to explicitly use either "El estado de Hawaii" or "La isla de Hawaii" as the reference (ground) object, or simply leave the expression ambiguous. On the other hand, German has two prepositions (**an** and **auf**) that both normally translate to "on". **An** applies to lateral adjacency, whereas **auf** has a meaning closer to "on top of". A German speaker could use **an** or **auf** to force different meanings in cases where an *English* speaker would have to use additional words or would have to tolerate ambiguity.

Observing these differences allows us then to deduce when people use one image schema, and when they might use another. In the above example, native speakers of German, English and Dutch appear to share an image-schematic differentiation which is manifested in their use of prepositions. In this case, the use of image-schemata becomes observable, that is, we have some observable facts that can be accounted for by the assumption that image-schemata are used in the proposed form. The occasional situations in which English and German seem to require different prepositions (such as the fact that a car is "*in* the parking lot" yet "***auf*** [=on] **dem Parkplatz**") apparently apply to modern situations in which different base nouns are used in compound names for ground objects. But using a different noun that forces another image-schema, a German speaker would say "**der Wagen ist im Parkfeld**" ("the car is in the park(ing) field", between the white markings that delimit a space) or "**der Wagen ist in der Parkzone**" ("in the parking zone"). We expect that image-schemata themselves will be common across linguistic and cultural groups, but their use will differ with those factors. (Image-schemata for languages other than English, or for other cultures, have yet to be examined in detail.) Studies are needed to establish cross-linguistic differences in the way that image-schemata are applied to various geographical situations.

4.2 Models of Space

The previous section discussed the image-schemata which appear to mitigate between mind, perception, and language. In this section, we review models of geographic and other spaces, and their relation to naive physics and to navigation and wayfinding.

4.2.1 *Models of 'Small-Scale' Space*

Downs and Stea (1977, p. 197) distinguished *perceptual*[4] *space*, studied by psychologists such as Jean Piaget and his colleagues and followers (Piaget and Inhelder, 1956), from *"transperceptual" space* that geographers deal with, and that we are focussing upon in this paper. They claimed that "the two scales of space are quite distinct" (p. 197) in the ways people perceive and think about them. Later in the book, Downs and Stea (p. 199) contrasted the terms "small-scale perceptual space" and "large-scale geographic space." At about the same time, Benjamin Kuipers (1978, p. 129) defined *large-scale space* as "space whose structure cannot be observed from a single viewpoint," and by implication defined small-scale space as the complement of this. The large-scale vs. small-scale distinction of Kuipers does not quite correspond to a geographic vs. non-geographic contrast, since as Kuipers pointed out, a high mountain viewpoint or an aircraft permits direct visual perception of fairly large areas. Nevertheless, we will follow Kuipers, and use the term *large-scale space* as he defined it, and *small-scale space* to refer to subsets of space which are visible from a single point. (We also note that there is risk of confusion with cartographic use of the terms small-scale and large-scale; representing a small-scale space on a fixed medium would use a rather large-scale map, whereas fitting a large-scale space onto the same medium would require a small-scale map.)

Our cognitive models of small-scale space develop from direct perceptions of our everyday world, dominated by a combination of visual inputs and the interactions of our bodies with the objects in that space. People are very good at processing the visual field, and at interpreting observed sequences of images, which are essentially two-dimensional at the retinal level, to be views of objects in a three-dimensional space. In fact, it has been claimed that "the visual system attempts to interpret all stimulation

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[4] We use the term *perception* strictly to mean mental reactions to sensory inputs in the presence of a stimulus. Perception results when we hear, see, feel, taste, or smell. This usage is consistent with the meaning of 'perception' in psychology and cognitive science. Under this usage, what some geographers call 'environmental perception' really should be called 'environmental cognition.'

reaching the eyes as if it were reflected from a scene in three dimensions" (Haber and Wilkinson, 1982, p. 25).

As noted above, bodily (sensorimotor) experiences with small-scale space also play a key role in the ways we build our mental models of such spaces. Lakoff and Johnson (Lakoff and Johnson, 1980; Lakoff, 1987; Johnson, 1987) claim that our spatial concepts for small-scale space largely are projected from human-body space (see also Couclelis and Gale 1986), and Svorou (1988) has shown that spatial terms themselves also often have bodily groundings. The ways in which the body interacts with objects allow us to recognize 'basic-level' objects such as 'chairs' by the age of about two years (see Rosch, 1973).

People naturally build cognitive models based on the way they perceive familiar objects behaving (reacting to forces) in small-scale space. The field known as *naive physics* (sometimes 'common-sense physics') deals with the ways in which people typically *think* that physical objects behave. For example, many people not trained in formal physics think that, when a person drops a ball while walking, the ball will fall straight down (McClosky, 1983). Of course, according to Newtonian or classical physics, the ball retains a forward motion component, falls in a parabola, and must be dropped before the hand is directly over a target in order to hit that target. Naive physics has associated with it concepts of distance, direction, connectivity, continuity, etc., which might be termed a 'naive geometry'.

Concepts of naive physics are of great interest not only as an aid to understanding the behavior of physical objects, but because they help us to effectively reason and deal with situations which are currently not tractable with the methods of classical physics. For instance, the behavior of lettuce and salad dressing can be modelled using the principles of classical physics, but the resulting formal system is so complex that it is not useful, for example, to guide a robot (Hobbs and Moore, 1985, p. xi). Principles of 'naive' physics may be successfully and easily used in such situations, and produce adequate results. By analogy, we expect that a formalization of some of the 'naive' geometric reasoning used in geographic space may be valuable for expert systems exploiting geographic data collections. Perception of the physics of everyday objects, together with our own bodily structures, also influences the way we perceive and label the structure of space. Gravity is so pervasive that the up-down axis is obviously the most *salient*, or most important to human perception and cognition. The horizontal plane, perpendicular to this vertical axis, is less differentiated in the environment. However, for humans, the front-back contrast, while less salient than up-down, is considerably more salient than left-right. This observation, discussed by Freeman (1975) and by many others, probably arises due to the fact that humans and most other animals show bilateral symmetry for most external components. This salience ordering of the three dimensions of everyday space (up-down >> forward-back >> left-right), and the fact that the latter distinction is necessarily egocentric, is important to the models discussed later in this paper.

Introduction of concepts of measurement, mathematics, and science, especially during the time of the classic Greek philosophers, made a formalization of geometry and physics desirable. School books tell us that plane geometry was first developed in Egypt to allow for land-ownership boundaries (the cadastre) to be re-established after the annual floods of the Nile. Abstraction of this practical formalization into a set of axioms is credited to Euclid. Euclidean geometry conforms by and large to the geometry which we observe in our everyday lives. Current school curricula instill upon the pupil the idea that Euclidean geometry is the only 'correct' geometry.

A formal theory of physics proved more elusive, and Aristotle's physics was fundamentally flawed. For example, Aristotelean physics predicts that an object must expend energy to keep moving, and will stop if force is no longer applied to it, but the fact that everyday objects behave this way is due to friction, and not the fundamentals of mechanics (see Di Sessa, 1982, for a discussion of Aristotelean, Newtonian, and naive physics). The classical physics which corresponds closely to the behavior of everyday objects in small-scale space is usually attributed to Sir Isaac Newton. Newtonian (solid-body) physics corresponds with naive physics well enough that people who 'believe in' Newtonian physics can deal with everyday objects as if the objects were governed by its 'Laws'. (For further discussions of naive physics, see McClosky, 1983, or Hobbs and Moore, 1985.) Newtonian physics conforms closely with observable reality, while at the same time is a highly abstract, formal system which is extremely useful in engineering and scientific applications, where it can be used to build models and to predict accurately the behavior of mechanical systems. Furthermore, Newtonian physics is completely consistent with Euclidean geometry.

4.2.2 *Models of Geographic Space*

The region of space that we can experience bodily at any moment is limited to a few cubic meters; the region we can experience visually usually is larger and much more variable. However, the combined extent of all the spaces that we experience during the course of a day's activities usually is much larger again. As noted above, Kuipers (1978, p. 129) called this large-scale space, defining this as "space whose structure cannot be observed from a single viewpoint." At some risk of criticism, we call this *geographic* space. Note that experience with this space is intimately intertwined with wayfinding and navigation.

Kuipers' model of spatial knowledge acquisition (Kuipers, 1978, 1983a, 1983b) begins from a sensorimotor experiential base. As we move through geographic space, we see a sequence of views (a 'view' is defined as the sum total of all sensory inputs when at a point and oriented in a particular way, but for most people, the 'views' are dominated by visual inputs). With some views, we associate actions; some actions form part of the navigation or way-finding process, and other actions relate to other activities. Kuipers' TOUR model (implemented as a computer program in LISP) uses as input ordered sequences of view-action (V->A) pairs. The routes form a 'spaghetti' of familiar paths, which constitute procedures for getting from one place to another[5]. Note that this kind of spatial knowledge is termed 'topological' by Piaget and his followers (Piaget and Inhelder, 1956), and 'procedural' by Thorndyke and Hayes-Roth (1982) and by Mark and McGranaghan (1986). Because these large-scale spaces are the ones that geographers often study, we consider 'geographic space' to be roughly synonymous with 'large-scale space'.

Kuipers (1978, 1983a, 1983b) noted that, as people find their way along various paths, they may recognize that the paths have some points ('places') in common. This allows them to use inference rules to build network models of places and connections, paths and barriers, in geographic space. Such a cognitive model of geographic space allows route-planning to novel destinations, or the planning of alternate routes when habitual paths are blocked. (Incidentally, such adaptive route-planning appears not to be restricted to human beings; Tolman (1948) discussed experiments in which laboratory rats were observed to use alternate paths when the usual ones were blocked by barriers.) Paths may have associated with them properties such as length in miles, kilometers, or blocks, or expected traversal times or effort, but global geometric properties, such as locations, straight line distances between points, cardinal directions, etc., often are weakly defined, inaccurate, or are absent from the model. Such properties of some cognitive models of geographic (large scale) space were noted very early by Trowbridge (1913).

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[5] We first used the spaghetti metaphor here because of the frequent use of the term 'spaghetti files' in digital cartography. However, in his work *The Songlines*, Bruce Chatwin explicitly used the 'spaghetti' metaphor in describing the models of geographic space that are central to Australian aboriginals' myths and traditions: "One should perhaps visualize the Songlines as a spaghetti of Iliads and Odysseys, writhing this way and that, in which every 'episode' was readable in terms of geology" (Chatwin, 1988, p. 16).

In Kuipers' TOUR model, spatial inference rules allow the model to be refined more and more, as more and more (V->A)-pair sequences are learned and assimilated, until a 'geometrically-correct' model of geographic space is built up. Such configurational models of space apparently are formed by at least some other organisms; for an example, see Gould's (1986) work on the 'cognitive maps' of honey bees. However, it seems that, for many people, such a two-dimensional Euclidean (cartesian) model of geographic space is never built from experience alone, or at least that it takes a very long time. Mark and McGranaghan (1986, p. 402) felt that "access to graphic, metrically-correct maps almost certainly plays a key role" in the development of a cartesian cognitive model of geographic space. Such a conjecture is implicit in the findings of Thorndyke and Hayes-Roth (1982), and is supported by recent experiments by Lloyd (1989a, 1989b).
Matthew McGranaghan has stated that the power of maps comes from the fact that they represent space with space[6]. In fact, maps represents use a *small-scale* space, namely a piece of paper or a computer screen, as a model of a *large-scale* (geographic) space. This allows people to experience some aspects of the geometry of a geographic space indirectly, but in a 'familiar' way, that is, the way they experience objects in small-scale space, as they experience objects on a desk-top or kitchen table in their everyday lives. Thus the map allows people to extend Euclidean geometry to geographic space, to be used as a basis for spatial inference, reasoning, and decision-making.

5. What is the 'Objective' Geometry of Geographic Space?

There is little doubt that maps allow people to extend the geometry of small-scale space outward to geographic space. Whether this is appropriate or not depends primarily on the use which is made of the geometry, and on how different the geometry is from the 'geometry' of perceived (experiential) reality. And the difference must be judged in the context of the specific task.

If one believes that Euclidean geometry is also the 'true' or 'objective' geometry of geographic space, then the map is a very valuable tool, since it allows us to grasp this 'truth' and use it. With a map in hand, or with a map-based cognitive model of space, one can plan routes and perform other spatial inference using the familiar Euclidean model. If, however, the perceived geometric properties of geographic space is not compatible

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[6] Paper presentation at the Eighth International Symposium on Computer-Assisted Cartography (Auto-Carto 8); the comment does not appear in the written version of his paper, which appeared in the proceedings of that meeting.

with the Euclidian geometry of the map, then the map may be an 'incorrect' model for geographic space. The map model of geographic space would be a sort of specification error. Road maps, navigational charts, and topographic maps present Euclidean views of the world, and are very useful. But the famous schematic of the London underground (subway system), and the other subway maps which mimic it, are also very useful, and most assuredly not Euclidean.

In light of this question about the relation between Euclidean maps and experiential space, one must wonder about the method used by Brody (1981) in his work on land-use and occupancy patterns for the aboriginal peoples in northwestern Canada. The Athapaskan informants were asked to draw their hunting, berry-picking, fishing, and trapping areas on topographic maps of a scale of 1:250,000. It seems unlikely that this procedure captured their concept of their space. However, perhaps the authorities would not have believed them otherwise:

"But when they discovered a sports hunter's equipment cache and an old campsite a few miles from the bear kill, their expressions of indignation were nothing if not political. As he uncovered cans of fuel, ropes, and tarpaulins, and looked around to see if a kill had been made, Atsin declared over and over again that white men had no right to hunt there, on the Indians' land. When Joseph [an Indian elder] heard about the cache he said: 'Pretty soon we'll fix it all up. We've made maps and everyone will see where we have our land." (Brody, 1981, p. 270)

Our examination of the concept of 'objective' or 'correct' geometry in this section have rested on an assumption that the 'real world' exists, and that it has 'objective' properties. This is an assumption and not a 'fact', since the human mind has no 'direct' access to the real world, but only is aware of what the senses appear to report. Since the decision to adopt a particular definition of objectivity is itself subjective, Hillary Putnam has shown that a paradigm of complete objectivity is internally inconsistent (see discussion in Lakoff, 1987, pp. 229-259). Nevertheless, experiential realism, discussed above, is based on the idea that there *is* a real world, which has consistent properties, so that when people interact with that world, their mental experiences are very similar.

One way to escape from this problem is to arbitrarily adopt a definition of objectivity. An obvious candidate, common in the sciences, is to declare that objective properties are those that can be *measured in a reproducibile way.* In that case, one could reasonably claim that 'the' geometry of surveying is the 'correct' and 'objective' geometry of geographic space. At scales ranging from planet Earth to the human body, Euclidean geometry and Newtonian physics seem to provide a geometry and physics

(respectively) which are mathematically formal, yet consistent with measurement and observation. The fact that Euclidean geometry breaks down at certain time, space, or velocity scales, and that Einstein’s theory of relativity required new geometries, thus re-orienting the cutting edge of academic geometry, is of little relevance to geography and surveying.
It is not far wrong to view our planet as a spheroidal solid body in Euclidean three-dimensional space; geodesy has established the shape of that body, and of the geoid. The surface of the earth is essentially a two-dimensional manifold stretching over the surface of that geoid; position can be denoted as two angles (latitude and longitude), and elevation above ’sea-level’ at any point may be defined as the height above that geoid. Geodesists and surveyors routinely use such a model and with the precision of the measurement techniques available today (generally better than 1 part in 10^6) do not observe any discrepancies between the model and their observations.

Map projections allow us to transform from one two-dimensional surface (over the spheroid) to another (a cartesian plane) in ways which control the geometric distortions that necessarily result. For ’sufficiently-small’ regions of the planet (say, up to about the size of the 48 contiguous states of the United States), the curvature of the planet can more or less be ignored; map projections exist which show almost no distortion of areas, angles, or distances over regions of that size or smaller (see Snyder, 1982).

In a scientific (positivist) view, measurement is often considered to be the only way to ’see’ space in an objective way. However, it also is possible to define ’correct’ in a way which does not rely on the concept of measurement. People usually experience space not by measurements, but rather by observing results of processes that are related to space. An every-day example for such a process is that physical movement in space requires time. Travel time and effort are usually proportional to the distance between two points, although the relationship is seldom linear.

On a conceptual level, the difficult task is to combine the multiple, conflicting concepts that people use in their interaction with objects in space, and to model how these concepts influence specific spatial behavior. Geography deals with many of these spatial processes, and thus geography and geographers can play a key role in discovering the spatial properties influencing these processes; this may in turn help researchers to understand human spatial cognition.

6. Spatial Cognition and Geographic Information Systems

Considerable effort has been spent over the last two decades to build geographic information systems (GIS). Numerous organizations have collected data and built GISs or other similar "spatial information systems". Not all of these systems have met with success. Many of the systems constructed were either extremely limited in their capabilities to exploit spatial location, or the methods used were mathematically well-defined but not necessarily 'intuitive', i.e. they did not agree with the spatial concepts used by all their users. The slow progress in GIS development appears at least partially to be due to the lack of formal understanding of spatial concepts as they apply to geographic space (see discussion under "GIS and Theoretical Geography", above).

In order for a GIS to be an effective information system and a useful tool for spatial analysis, the concepts it embodies and the ones employed by its users must be as similar as possible. This similarity can be achieved by training the user to understand the concepts used by the system. But such a strategy requires a great deal of training, and thus may severely limit the user community and thus the applicability of the system. Alternatively, the system can be built using concepts very close to the ones that an untrained user would expect. Current systems are primarily designed and constructed following the first approach.

In the preceding sections, we discussed some observations regarding concepts people use to structure geographic space. A GIS should reflect these concepts, and particularly that the user interfaces for such systems should be 'natural'. In the remainder of this section, we will discuss the mathematical bases of geometry, and how such concepts could be formalized. Unless these concepts can be formalized they cannot be included in a GIS; but conversely, the inclusion of any of these concepts in a GIS implementation may constitute the required formalization.

A GIS is a fixed set of instructions embodying in a formal way a set of procedures (algorithms) to process data. To develop such programs properly, a clear, formal, theoretical base is necessary. Most GISs are based on Euclidian geometry and implemented using analytical geometry: every point or line is situated on a coordinate plane, and the locations of the points are characterized by coordinate pairs. The assumption is that all other necessary or interesting spatial properties can be derived from these points and their coordinates. Euclidian geometry and the formulae of analytical geometry are well known and relatively easy to understand, and thus the actual writing of a GIS was expected to be an easy and straightforward task.

There are however, a number of problems related to the use of Euclidian geometry in this manner. First, the implementation of GIS concepts as a computer program is not straight forward. Analytical geometry and the validity of its formulae assume a coordinate plane created from real numbers (R x R). A computer, being a finite-precision system, cannot implement real numbers exactly, but only can represent approximations of them. These approximations are limited both in their magnitude (over- or under-flow conditions arise if results of computations become too large or too small) and in their resolution. In Euclidean geometry, one can always find an intermediate point exactly half way between any two given points; in computer coordinates, however, this is often not possible. Known GIS implementations show surprising artifacts that are due to this problem; they may even break down in unexpected situations (see Franklin, 1984). The implementation of analytical analytical geometry on a computer is really a geometry on a discrete (though admittedly very fine) grid, where point locations are restricted to grid points. In such a situation, many of the standard laws of Euclidian geometry do not hold (Franklin, 1984; Nievergelt and Schorn, 1988).

A GIS programmer thus faces the problem of taking conceptual framework expressed in Euclidean geometry, and expressing it as a program on a finite-precision digital computer (Figure 1, right side). But if GISs are to reflect the concepts that untrained users might employ, the software engineer designing the GIS must transform the naive geometry of the user into the (quasi-)Euclidean system that the programmer can implement (Figure 1, left side).

Naive Geometry ----> Abstract Euclidian----> Geometry of Implemented
of the User Geometry GIS

Figure 1: Using Euclidean geometry in modelling geographic information on computers involves two transformations.

The concepts applied by users of geographic information are not exactly Euclidian. This is not so much a disagreement with the concepts that Euclidian geometry proposes, but rather involves additional concepts that are not included in Euclidian geometry (for example, the direction between extended objects). GIS the are occasionally unable to answer questions which appear reasonable and well defined to the user, like ’What is the direction from New York to Canada?’. Peuquet and Zhan (1987) investigated this problem, and provided solutions for some situations.

It is unlikely that the shortcomings of the one mapping can be compensated for by the other. More likely, the problems will be compounded, and the user will be forced to learn how to transform his concepts into the Euclidian geometry, and may be surprised to see that the implementation is not following the theory that he has just learned. A more sensible solution would be to directly map the user concepts to the implementation, bypassing Euclidian geometry (see Figure 2). But such a mapping is far from trivial, and requires groundwork in cognitive science.

Naive Geometry --?-> Geometry of Implemented
of the User GIS

Figure 2: If a GIS users’ naive geometry can be modelled directly on the computer, a more successful implementation should result. However, it is not yet known how to accomplish this.

A second problem in achieving working GISs is caused by the fact that geographers often work with data that are the results of measurement and processing, and contain an error component. People are accustomed to the common, small positional errors one encounters when combining data from different sources. Euclidian geometry on the other hand assumes ideal points and lines and locations without error.

7. Formalization of conceptual geometries

In order to implement the concepts that users may have regarding space and spatial relations, the concepts must be formalized, that is, converted into a formal mathematical theory. This presumably will lead to a new and different geometry. Constructing new geometries is not unheard of in mathematics. Until the beginning of the last century, Euclidean geometry was the only form of geometry. Efforts to show that his set of axioms was minimal, and especially to show that the axiom based on parallel lines was independent of others, led to the discovery that other geometries were possible, and indeed that their construction is straightforward. Hyperbolic (or Lobachevskian) geometry, where two lines can fail to intersect and yet still not be parallel, was constructed by replacing Euclid’s axiom for parallel lines with its negation. The elliptic (or Riemannian) geometry, in which no two lines are parallel to each other, is another geometry, and one which has a well known application: the geometry on the sphere is (double) elliptical, and any two ’lines’ (great circles) intersect in two points (Blumenthal, 1986, p. 176). Although it was difficult for some mathematicians and scientists to accept the fact that there were other geometries in addition to the widely accepted one, these new concepts of geometry became extremely important for developments in physics, especially the theory of relativity. Despite the fact that non-Euclidean geometries have been discussed in geographic contexts by Harvey (1969, pp. 199-203), Tobler (1976), Müller (1982), and others, they have not made inroads into mainstream geographic models or (especially) into geographic information systems.

For mathematicians, however, the problem was not testing whether a particular geometry was useful or not. Instead, as more that one geometric theory was designed, the problem became the determination of what made a theory of space a ***geometry***. What is the essence of a geometry? Felix

Klein, in his famous 'Erlanger program' (Klein, 1872), which influenced the development of mathematics for several decades, defined the field of geometry by a concern for properties of objects which remain unchanged (invariant) when the object was subjected to one of a group of transformations. A transformation in this case is defined as a mapping of a space onto itself. For example, Euclidian geometry deals with properties such as the length of a line, the sizes of angles, etc., all of which remain invariant under the transformations of rotations and translations. This definition of geometry also includes areas of mathematics such as graph theory and topology, which have a geometric component. Typically, each group of transformations defines a set of properties which remain invariant and thus creates a geometry which can be formally defined and studied.

This definition of geometry, based on groups of transformations, reflects a similar structure to that which we see in Talmy's pragmatic approach to linguistic representations of space (Talmy 1983, p. 258 - 263; see also Talmy 1988, p. B-3). It seems more appropriate for our purposes than some further refinements, which replace the group of transformations by equivalence classes (Blumenthal and Menger, 1970, p. 27). The use of groups of transformations is also part of the method used by Couclelis and Gale (1986), when they studied invariants of movements.
This more general framework for the definition of a geometry is well-suited to questions such as: "what is the geometry of natural language?" And perhaps this should be: "what *are* the geometr*ies* of natural language?", since there is evidence to suggest that there is more than one such geometry (see Couclelis and Gale, 1986, for a discussion based on formal properties). Expressions of spatial relations and properties in natural language are typically invariant, in most languages, under a wide set of transformations (Talmy, 1983; Talmy, 1988). As an example, the English-language preposition 'in', representing the CONTAINER image-schema, apparently is:

- *material neutral*: (the use of 'in' is independent of the materials from which the figure and the ground are composed);
- *magnitude neutral*: ('in' is used without regard to the size of the figure or the ground);
- *shape neutral*: (the shapes of the figure and the ground are irrelevant);
- *closure neutral*: (the preposition is used whether the ground is completely closed [as in a box] or partially open [as in a bowl]); and
- *continuity-neutral*: ('in" is used both for continuous enclosures, discontinuous enclosures [such as a bird cage], or conceptual (e.g., 'in

town', or even 'in love').

And, whereas some languages have a few examples that depart from this pattern, these independences seem to be the rule in natural language.

Klein's mathematical definition of a geometry is similar to the invariance concepts encoded in natural languages: both identify properties that are invariant under transformations. (Note, however, that as we extend Klein's concept to natural language, we must relax Klein's requirement that the transformations form a mathematical group.) Not all terms of a natural language define the same geometry, as they may remain invariant under different sets of transformations (i.e. appropriate sets of transformations will define geometries, each of which will include some of the spatial relationships expressed in natural language). For example, all properties expressed in a reference frame which is bound to the referent are invariant under translation and rotation (of the object and the referent). No matter which cardinal direction a church faces, a nearby cemetery will almost always be referred to as being "behind the church" if it is near the wall of the church that is opposite the main entrance. Properties expressed in absolute reference frames are only invariant under transformations which leave the reference frames invariant (for example, an expression using cardinal points would be invariant under a translation, but not for a rotation).

Comparing mathematical theory and linguistic observations raises a number of interesting questions. Reference frames have been well-studied in linguistics. Of particular interest are situations which are quite different from 'standard geometry' and do not depend on cardinal directions (astronomical reference frame) but use, for example, a radial system, as is customary on many islands (for the example of Icelandic, see Haugen, 1957; for a partial review and discussion, see Mark, Svorou, and Zubin, 1987) and in some circular lakes.

The last step in the development of geometries of natural language(s) will be to bind these geometries into a comprehensive system, in which the properties of features can be from any of the different geometries. A need for such a scheme is already manifest in the efforts to combine raster and vector-based data in GIS. The same problem is also manifest in organizations that maintain multiple databases which contain the same features but at, for example, different levels of resolution or different levels of generalization (Buttenfield and DeLotto, 1989). Current systems not capable of managing such collections of data as single logical units, in which changes propagate from one level to the other and queries are executed in the most appropriate representation of particular features.

Research has begun to address this general problem. A new, promising approach is based on the use of algebraic descriptions of each of these geometries, the traditional mathematical ones as well as formalization of the conceptual ones. There is substantial methodological knowledge of how multi-sorted algebras (Birkhoff and Lipson, 1970) can be used to describe objects and the system of operations associated with them. The method is extensively used in software engineering and is known as object-oriented specification (Guttag, Horrowitz, and Mousser, 1978; Goguen, Thatcher, and Wagner, 1978). It has already been applied to geometric problems (Goguen 1988; Mallgreen, 1982). Algebraic specifications for cell complexes have been advocated for use as the base modelling block in a vector oriented GIS (Frank and Kuhn, 1986; Bruegger and Frank, 1989).

Given such individual algebraic specifications for a specific geometry, we have then to construct relations between them. Mathematicians have studied the connections between different algebras under the topic of algebraic morphism. They establish mappings between the objects in the one algebra to the ones in the other and map operations from one algebra to the other. Then one can study the regularities in these mappings. A homomorphism between an algebraic structure with elements **A** (a1, a2, ...) and operations **f** and another structure with elements **A'** (a'1, a'2...) and operations **f'** is a mapping **G** which maps elements from **A**to **A'** and also maps the operations **f** to **f'**. The mapping **G** is said to be a homomorphism if **f**(ax)' = **f'**(a'x), meaning we can go from **A** to **A'** first and then apply the operation **f'** or first apply **f** and then go to **A'**. Computation with logarithmic values provide a practical example for an application. Consider the mapping 'logarithm' from positive real numbers to real numbers. This establishes an isomorphism between $(R^{+},*)$ and (R, +), mapping multiplication to addition, due to the equations:
$(a^{b})(a^{c}) = a^{(b+c)}$
and
$\log(a) + \log(b) = \log(ab)$

Mathematicians have used this isomorphism to replace difficult multiplications by simple additions of the logarithmic values. There is an extensive theory about such morphism, called category theory, which might be applicable here (Geroch 1985). This approach based on isomorphisms can and will be used to construct formal relations between the points, lines, and areas of cartographic data structures and Euclidean geometry, and the new geometries of natural language and cognition.

8. Summary

Development of a comprehensive model of spatial relations and properties is important for the future development of systems for geographic information and analysis, and also for cognitive science and behavioral geography. This paper first reviewed concepts of space. A critical distinction was made between small-scale spaces, whose geometry can be directly perceived through vision and other senses, and geographic space, which can be perceived only in relatively small parts. Fundamental terms for spatial relations are often based on concepts from small-scale space, and are metaphorically extended to geographic (large-scale) space. Thus, terms and concepts for the spatial relations among the objects in a small space can form an appropriate core for spatial language. Additional spatial relations on a geographic scale can be formed by the addition of small sets of axioms or postulates (for example, letting "north" equal "up"). Finally, we set as a short-term but important goal a search for geometries of spatial language. This search will attempt to define those properties of particular instances of spatial reference in natural language which remain invariant under groups of transformations, and the development of a link between these properties and the geometry and topology of GISs. This fusion could form the basis both for geographic data structures and for the understanding and generation of spatial language itself. If properly formalized it will be an effective base for constructing GIS software which will be more 'natural' to use.

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