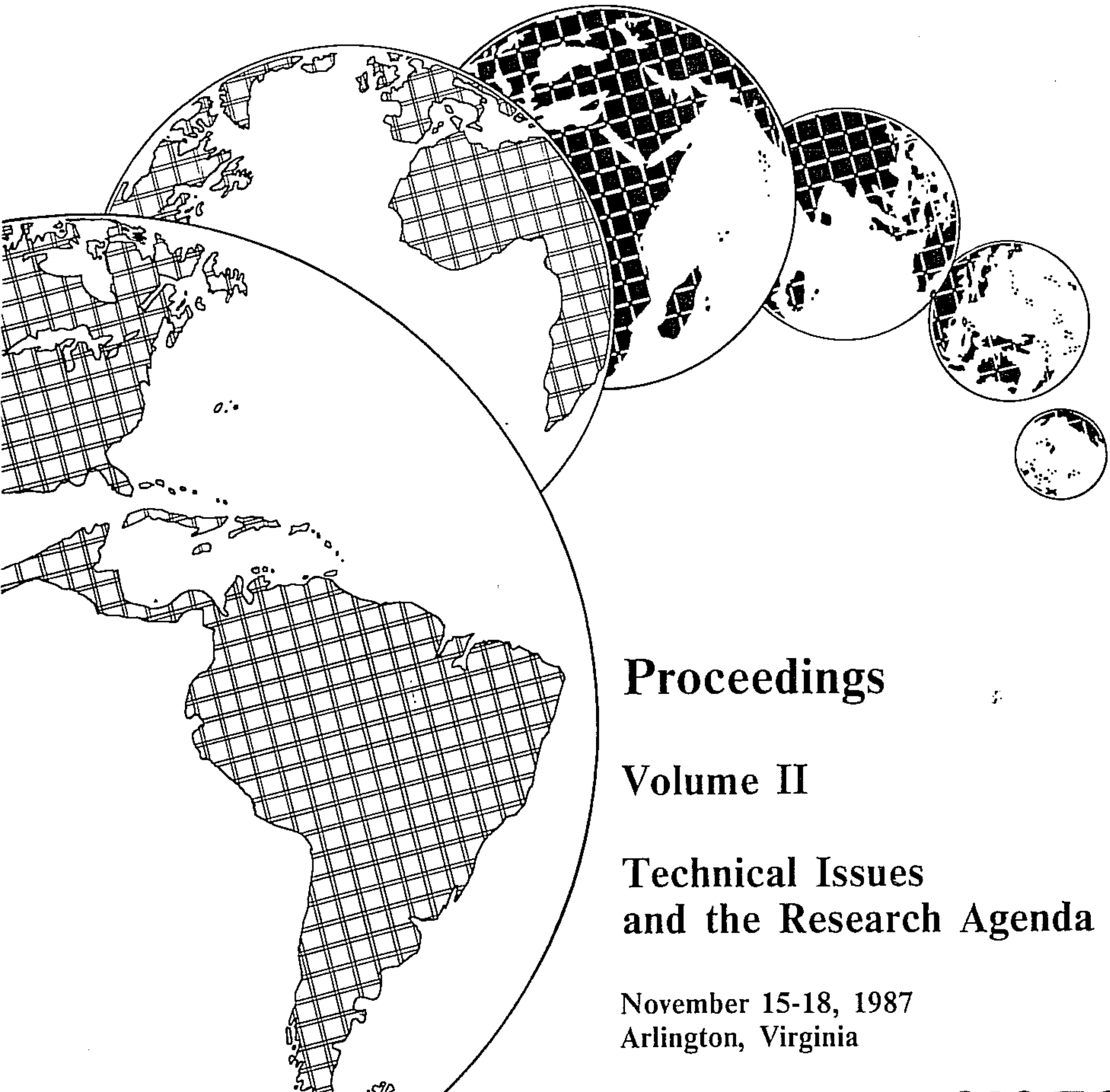
International Geographic Information Systems (IGIS) Symposium: The Research Agenda



Proceedings

Volume II

Technical Issues and the Research Agenda

November 15-18, 1987
Arlington, Virginia

Towards A Spatial Theory*

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Abstract

In several recent publications, problems with spatial reasoning and the lack of a coherent spatial theory have been noted. Human beings are very good at analyzing spatial situations, determining patterns and similarities, etc.; however, programmed systems on computers cannot compete.

The well-known Euclidian geometry is not a true image of how human beings think about geometry. The ideal objects of Euclidian geometry are quite unlike the real, extended objects of which the world is made. Moreover, Euclidian geometry cannot be easily modelled on computer systems. The absence of a coherent spatial theory hinders development of GIS. Building a comprehensive, coherent spatial theory will not be simple. Several different approaches are currently tried: (1) defining algebras with finite representation that can model Euclidian geometry, (2) using combinatorial topology to represent geometry with simplices, and (3) applying models based on finite resolution rasters.

It is important that multiple approaches are tried and ultimatetly combined to best accommodate the concepts human beings use. In cognitive science, efforts to better understand spatial concepts will be contribute to a spatial theory.

1 Introduction

The development of computer systems to the present level of performance and reduced cost make many applications possible which were not conceivable a number of years ago. Computers can now be used to perform operations that in the past were carried out tediously by hand. We can also see new applications which were not

*This work was partially funded by a grant from NSF under No. IST-8609123

possible with the traditional technology. This development does not only influence the technical solutions we select or the problems we can successfully attack, but it will also influence the scientific concepts we use [Cebrian 1987]. This paper deals in the foreground with practical problems and possible technical solutions, but the methods proposed can be used to further the theoretical understanding of spatial situations and processes.

Spatial information systems, used as the common name for geographic information systems and land information systems [FIG 1981] are in a way a combination of traditional tasks and new concepts. Several decades ago, the possibility for this development became visible and a large number of optimistic predictions have been written; however, an assessment today reveals that only few system are productive and their use, although beneficial, is much less general than thought possible.

The development of software for spatial information systems falls behind user demands and some of the available programs still show major shortcomings. For example, none of the commercially available GIS software systems contain an algorithm for polygon overlay which is absolutely foolproof. The best ones work in practical cases, but some fail even with relativly simple, practical problems. A recent discussion with a development team confirmed that the programming of geometric algorithms is the major difficulty. It was generally believed that the geometric operations were simple to program—after all, "this is (2-dimensional) high school analytical geometry". Unfortunately, this is not correct and we begin to understand our errors. We stil do not completely understand the tasks that human beings and computers do well. Presentlt, human reasoning exceeds the computer's ability to analyse and deal with spatial situations. This paper attempts to outline a research program to understand the human capabilities better, and find possibilities to model them in computer systems.

Too often computer geometry and computer graphics have been lumped together. In this report we separate the concerns of processing geometric data from considerations for output presentation. Clearly, graphical presentation methods are based on geometric processing, but may have different requirements than processing of geographic data for analytical purposes.

As a second level of separation, we advocate the analysis of geometric aspects of information in a separate step from the treatment of the associated attribute data. Clearly a spatial information system must contain routines to treat both classes of data, but a separation of concerns will simplify the design and coding [Frank 1987]. This paper deals with the spatial aspects only.

2 The Concept Of Spatial Theory

Boyle [Boyle 1983] reports on a NASA sponsored conference about the "review and synthesis of problems and directions for large scale cartographic information system development." The conference report contains an extensive discussion of the need for a spatial theory—a term which was first used by Chrisman [Chrisman 1975]:

> "There is at present no coherent mathematical theory of spatial relations.
>
> The world in which we live is a spatial world and to exist in it all of us master spatial relations on an intuitive basis. While these relationships are highly complex, we usually need not even think about them. We make use of the eye/brain combination to sort out the visual images we receive and recognize their various elements.
>
> In dealing with spatial data in the form of maps, images and the like, we usually obtain the data with our eyes and then deal with spatial aspects of the data on an intuitive basis.
>
> Unfortunately, when a computer must be instructed in how to perform such operations, intuition must be replace by precise statements and by precise mathematical relationships.
>
> At present we cannot supply this need.
>
> The lack of a coherent theory of spatial relations hinders the use of automated geographic information systems at nearly every point. It is difficult to design efficient data bases. Difficult to phrase queries of such data bases in an effective way, difficult to interconnect the various subsystems in ways which enhance overall system function, difficult to design data processing algorithms which are effective and efficient. As we begin work with very large spatial data bases or global data bases the inabilities and inefficiencies which result from this lack of theory are likely to grow geometrically.
>
> While we can continue to make some improvement in the use of automated geographic information systems without such a coherent theory on which to base our progress, it will mean that the development will rest on an inevitably shaky base and progress is likely to be much slower than it might be if we had a theory to direct our steps. It may bee that some advances will simply be impossible in the absence of a guiding theory." [Boyle 1983]

A spatial theory must therefore include the following points:

- Define a set of generic geometric objects and generic operations on them. Special attention must be directed to include special cases and error conditions.

- The generic objects and operations must be suitable for modelling the geometric properties in a spatial information system; that is, the higher level objects are distinguished by their attributes which restrict and redefine the geometric operations applicable. The quality of the theory is judged by its capability to reduce redundancy and possible inconsistencies in the definiton of specific geometric objects.

- Explain the behavior of commonly used operations on geometric objects in terms of the previously precisely defined objects and operations. The success of these definitions can be measured if the commonly used operations can be described easily.

- A spatial theory should be as general as possible, thus ensuring a wide range of applications. In the following proposal, we seek for a theory which is independent of the dimensions of the objects it is applied to (i.e. it can be used for the two or the three dimensional case).

2.1 Need For A Spatial Theory

A coherent spatial theory should tie together the different concepts of space which are used by the builders and users of a spatial information system. The differences in concepts between the mathematically inclined, computational geometry research and the practical users of systems are wide, and the gap is difficult to bridge. The differences appear in the user interface where users are surprised by system reactions because the concepts they use and the spatial concepts embedded in the system differ substantially. Building comfortable interfaces is only possible if a clear understanding of the user concepts is available, or a set of coherent concepts, which are used for the design of the interface, can be taught to the user. In order to have the system perform, the operations on the user level—and such operations may be different for different classes of applications—such concepts must be general enough that they can be used internally for the design of the basic geometric software. The development of geometric software is so difficult and thus expensive, that a generalized solution can be developed which is useful for spatial information systems as well as for all physical CAD/CAM systems for mechanical and civil engineering.

Clear definitions for spatial concepts are also crucial for data exchange. Transfer of data from one system to another is difficult if the model used for spatial representation differs significantly. A standard for exchange of spatial data needs a formal definition of spatial concepts which can be used to establish conversion routines.

2.2 What Spatial Theories Are Available

A number of theories deal with space from different points of view. We often revert to the high school form of Euclidian geometry and very few know of "elementary geometry from an advanced standpoint." [Moise 1963] Mathematicians have further studied topology, including graph theory (which is often used in spatial information systems). Analytical geometry shows how to transform geometric objects and operations to the realm of computations (and computers).

Geodesy deals with some aspects of physical space and how they can be measured precisely, whereas GIS often uses rasterized methods to approximate the spatial distribution of some properties. Physics, in general, includes some aspects of the geometric properties of objects—the recent development of "naive physics" [Hobbs 1985] attempts to capture some of the common understanding of geometric properties of reality with less abstraction than is evident in the pure theory.

3 Classical Geometry vs. "Naive Geometry"

Geometry deals with abstract objects, like infinitely small points, lines which extend into infinity, etc. Physical objects in the real world are very different: points have a finite size, lines are finite and never truly straight or parallel. Moreover, our abilities to measure are limited and only approximations for the true values can be determined.

In this section, first the concept of geometry are discussed as they are seen by classical and modern mathematics before extending it to take into account the restrictions of real physical objects.

3.1 Classical View Of Geometry

Classical geometry, as developed by the Greeks and usually taught in high school, deals with abstract objects (primarily points and lines) and their interaction. All operations are restricted to be performed, at least in principle, by compass and straight edge.

These abstractions have been very helpful for a simple axiomatization of geometry, but it must be remembered, that there are no objects in reality like points and lines, or relations like parallel.

3.2 Concept Of Geometry

Geometry in modern mathematics is broader than Euclidian geometry. The aim is to determine the abstract concept of geometry. Since the work of Felix Klein [Klein 1872] it has been accepted that geometry deals with invariances of figures under transformations. This notion seems to capture more of what humans understand by "geometry" than the classical operations with points and lines. It is evident, that not only the figures 1a and 1b are related, but most will also see a relation between figures 2a and 2b. The figures 1 are related by similarity transformations (translations, rotations and scaling), whereas the figures 2 are related by more general topological transformation.

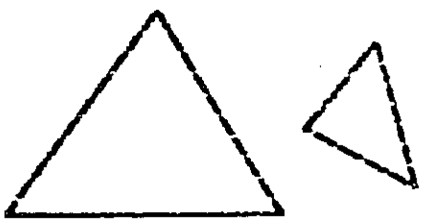

Figure 1 (a) and (b)

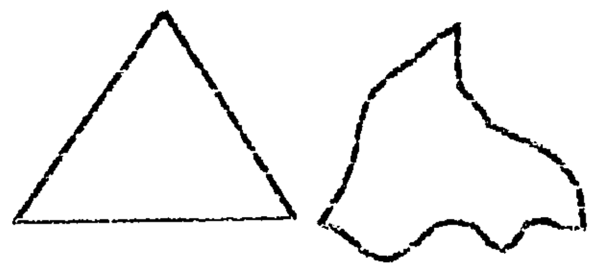

Figure 2 (a) and (b)

In a spatial information system we often want to search for similar patterns, but do not expect to find the same geometric figure again. A number of geographical studies, for example in migration analysis or geomorphology, try to find similar spatial patterns. It is an important task for a spatial theory to explain what "similar" means in each of these cases. There have been attempts to define patterns by using symbolic processing based on a triangular surface [Frank 1986]. The class of continuous transformations in mathematical geometry is too broad and limited. Sometimes, we judge two figures, which are linked by a continuous transformation, as quite different (figure 3a and 3b), or we judge two figures as similar, even if they are topologically different (figures 4a and 4b) [Walker 1987].

Figure 3 (a) and (b)

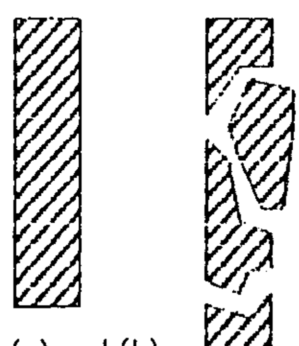

Figure 4 (a) and (b)

3.3 Naive Geometry

The objects we deal with daily are quite different from the abstract geometric concepts we encounter in the mathematical treatment of geometry. The extension of physical objects are limited, but most geometric concepts (e.g. line) are infinite. Secondly, geometry is based on the ability to measure without error, but all our measuring methods are inherently imprecise to a certain degree.

Our daily operations are not severely limited by these problems. As human beings we use a sort of 'naive' geometric reasoning, taking into account automatically the differences between abstract geometry and the geometric properties of real objects. Despite the fact that we all know how to do this, there is no formal understanding of this process. A formal description of this "naive" reasoning is necessary in order to have computers simulate such behavior. It seems worthwile to study if "fuzzy logic" [Zadeh 1981] applies to our reasoning about spatial relations ('close', 'next to', 'far', etc.).

In the arena of expert system construction, a similar disagreement was found between the laws of theoretical physics (i.e. what was learned in college in Physics I and II) and the concept humans apply in daily life—we take into account all the impurities of friction in motion without much trouble. The attempts to formalize this "naive physics", so as to be incorporated into an expert system has been difficult, but important in areas like building expert systems to assist in the diagnosis of faults in mechanical systems (e.g. railway engines).

A spatial theory should include a "naive geometry" in the above sense. It must be expected, that this would also be helpful for the development of the "naive physics", because much of physical reasoning is based on geometry.

4 Mathematical Geometry vs. Modelling Power Of Computer Systems

Modelling of geometry in computers is most always attempted based on analytical geometry. Analytical geometry provides a convenient base for translating geometric problems into numerical ones, thus making them more suitable for computer processing.

Unfortunately, the use of analytical geometry for computer modelling of geometric figures has some serious shortcomings. Analytical geometry needs a mapping to real numbers. The finite approximations in a computer system are not sufficient. The floating point numbers (REAL) of a FORTRAN program are much more akin to the integers: there is only a finite number of them and between some of the floating point numbers, there are no others.

Mathematically speaking, there is a difference between the field of real numbers and the integral domain (a ring). The Euclidian concepts of geometric space are intrinsically linked to the properties of spaces built over real numbers. It was shown, that analytical geometry using computer approximations is not in every case invariant under translation and rotation—the minimal requirements in Euclidian geometry [Franklin 1984].

5 Raster Based Systems vs. Vector Systems

Many useful spatial information systems are not based on a geometric description of objects with vectors, but use a regular subdivision of space in a raster and then record the interesting properties for each raster cell.

Raster systems are based on a intrinsically different concept of space then a vector system. In a vector system space is the total of the extension of all the objects, each with its own identity, special form, and properties, and the space is organized as a function of these objects. In a raster system, space is an abstract concept which is subdivided in abstract cells, for which we know some properties.

In a raster system, objects can be approximated by a set of raster cells, but a raster system can not deal with the abstract geometric objects like points and lines, and must approximate them with small areas. Typically, line oriented operations are not possible, but a number of other interesting operations (shrink & grow) are available. This is primarily studied in image processing literature. In [Serra 1982] an algebraic treatment of typical raster operations (transformations) is proposed.

6 Spatial Relationships

In general we are not only interested in spatial objects, but also in the spatial relations between them. Again, we lack a comprehensive, formally defined set of spatial relationships.

6.1 Distance and Direction

In Euclidian geometry (generally in a metric space) there is a distance defined between any two points with the ordinary properties:

- $d(a, a) = 0$
- $d(a, b) = d(b, a)$
- $d(a, b) + d(b, c) \geq d(a, c)$

In an Euclidian space (generally in any vector space) there is also a direction defined.

Both of these operations are defined between two points. It is interesting to see, that distance and direction are not defined in the same type of space and are thus not invariant under the same transformations; however, both obey linear transformations to be made compatible with rotation or scaling.

Human beings use distance daily between extended areal objects to discuss the direction between two areas (Canada is north of Maine? - it is also to the east and to the west). It seems possible to treat distance and direction between extended objects using intervals and then apply interval relations which are closely related to the inclusion relations between extended objects. An obvious first idea is to replace the two objects by their "center" (e.g. their center of gravity) and then determine distance and direction between these. This does not adequately represent human beings' concept in a large number of cases. Peuquet proposes a method based on heuristics to deal with distance and direction between extended objects, which avoids the most obvious shortcomings [Peuquet 1987]. We experiment with a system based on an algebra of intervals.

6.2 Boolean Operations

For objects in a topological space, an intersection operation is derived from set intersection. It is possible to determine if an object is spatially included in another, if they overlap or are disjoint. It is also possible to define multiple kinds of "touching" between objects; moreover, a complete formalization has not yet been achieved.

6.3 Natural Language Concepts For Spatial Relationships

Natural languages use a number of concepts to express the relationships between objects in space. Investigations are just beginning, which analyze the concepts and separate methods, specific to some languages, and others which seem universal [Mark 1987].

In natural language, human beings use concepts which are not precisely defined, like "far" and "near". Their interpretation clearly depends on the context of their usage (e.g. "near" applied to a gas station is different from "near" applied to a major airport), but are not intended to be precise even in a single context. It will be difficult to capture these effects and model them. Fuzzy logic [Zadeh 1981] was used to observe the usage of "near" and "far" in a set context [Robinson 1987].

We can assume that the concepts used to express spatial relationships in natural languages are the 'natural' concepts for human beings. It is important to understand them, formally define them and use them in at least two contexts: as key words in query languages and in system where the human being receives guidance to find a location from an information system.

6.3.1 Query Languages

Useful GIS must provide an interactive facility to ask questions to the system and see immediately answers in the form of small maps. In this context, there must be a language to express the desired information, including spatial relationships between objects, in order to specify the desired objects in a query to a GIS (e.g. show all the camping areas north of Bangor). In order for such a facility to work, the human understanding of the spatial relationships used, and the formal definitions applied by the system, must be very similar, otherwise the system will not retrieve and display the desired information, This may lead to frustration of the user if detected, or to erroneous judgement by the user if not detected.

6.3.2 Navigation Aids

The use of GIS to help humans navigate in complex situations seems to be an obvious application. Experiments with systems to navigate in towns to find locations specified by street address are underway, and the first commercial systems are available. It is expected that most cars in the future will include some navigational aid.

In such a system the user must be informed about spatial actions ("turn left at next red light"). It is not yet clear what type of concept for expressions of spatial relationships is most effective in this situation.

7 Possible Approaches

In this section we rapidly enumerate a number of approaches that are currently tried without assessment.

7.1 Other Number Systems

The work currently done at the Renselear Polytechnic Institute by W. Randolph Franklin attempts to overcome the problem that real numbers used in finite computer systems cannot correctly represent geometric situations. Rounding of computed coordinate values cannot be avoided. They study other number systems, e.g. (big) rational numbers [Franklin 1987] or number systems including roots of polynomials, which could overcome these limitations [Franklin 1984].

7.2 Raster Systems

Operations in current raster systems are more based on opportunity than on mathematical analysis. A most extensive treatment is found in [Tomlin 1983], but a more compact notation of the algorithm could show similarities in the proposed operations. Approaches to build an algebra of raster operations are shown [Serra 1982].

7.3 Combinatorial Topology

The application of combinatorial or algebraic topology to GIS has a long tradition [Corbett 1979]. Combinatorial topology offers concepts to deal with the topological relations between spatial objects in a symbolic form. By using a restricted form of a simplicial complex, we found that qualitative aspects of most of the metric properties can be included [Frank 1986a].

8 Conclusions

Understanding spatial situations and processes is a primary goal of most studies in geography. We clearly need a formally defined vocabulary that allows us to discuss and reason on these processes. By spatial theory we understand a set of concepts of abstract space and abstract operations on space, which are formally defined; however, such concepts are only valuable if they correspond to human beings' understanding of space. It is thus necessary to combine the formal theoretical studies with results from cognitive scientists, who analyze the 'natural' concepts we use.

Acknowledgement

Thanks to Jeff Jackson and Max Egenhofer who helped prepare this paper.

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