Qualitative Spatial Reasoning with Cardinal Directions[1]

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Abstract

Following reviews of previous approaches to spatial reasoning, a completely qualitative method for reasoning about cardinal directions, without recourse to analytical procedures, is introduced and a method is presented for a formal comparison with quantitative formulae. We use an algebraic method to formalize the meaning of cardinal directions. The standard directional symbols (N, S, E,W) are extended with a symbol 0 to denote an undecided case, which greatly increases the power of inference. Two examples of systems to determine and reason with cardinal directions are discussed in some detail and results from a prototype are given. The deduction rules for the coordination of directional symbols are formalized as equations; for inclusion in an expert system they can be coded as a look-up table (given in the text). The conclusions offer some direction for future work.

1. Introduction

Qualitative spatial reasoning is widely used by humans to understand, analyze, and conclude about the spatial environment when the information is qualitative; for example, received through a verbal channel. A formalization and computer implementation for qualitative spatial reasoning is necessary to understand spatial information expressed in natural language, is generally useful for user interfaces to spatial information systems (e.g. Geographic Information Systems), and is useful for optimization of spatial queries and for inclusion in expert systems.

Most methods of spatial reasoning translate the given problem from the quantitative to the qualitative realm and use analytical geometry to find a solution. This is not always a workable solution. The treatment of the inherent uncertainty in qualitative spatial descriptions creates problems. The construction of expert systems that deal with space and spatial problems has been recognized as difficult (Bobrow, et al. 1986). Here, a strictly qualitative approach is proposed and algebraic methods are used towards a formalization. The major step is the

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introduction of an identity symbol 0, that allows for a definition of rules which yield answers for all input values.

This paper is restricted to the specific problem of reasoning with cardinal directions. The problem addressed, described in practical terms, is the following: given the information that San Francisco is west of St. Louis, Chicago is north of St. Louis, and Baltimore is east of St. Louis, one can deduce the direction from San Francisco to Baltimore by the chain of deductions as follows:

1. Use 'San Francisco is west of St. Louis' and 'Baltimore is east of St. Louis', to establish a sequence of directions San Francisco - St. Louis - Baltimore .
2. Deduce 'St. Louis is west of Baltimore' from 'Baltimore is east of St. Louis'
3. Use the concept of transitivity: 'San Francisco is west of St. Louis' and 'St. Louis is west of Baltimore', thus conclude 'San Francisco is west of Baltimore' .

This paper formalizes such rules and makes them available for inclusion in an expert system.

The description of directional relationships between points in the plane can be formulated as propositions 'A is north of B' or 'north (A, B)'. Given a set of propositions one can then deduce other relative positions as the induced set of spatial constraints (Dutta 1990). Following an algebraic concept, one does not concentrate on directional relations between points but rather finds rules for the manipulation of the directional symbols themselves, when combined by operators. An algebra is defined by a set of symbols that are manipulated (here the directional symbols), a set of operations and axioms that define the outcome of the operations.

The two operations considered here are *'inversion'* and *'combination'* of paths. Directional symbols with the operations defined have properties very similar to the properties of algebraic groups, if one introduces an additional symbol for 0 (identity), such that $a \infty 0 = a$. This symbol can be interpreted as 'two points are so close that one cannot determine a direction'.

Cardinal directions are similar to the four directions 'front', 'back', 'left', and 'right', which are regularly used in spatial references (Herskovits 1986, Retz-Schmidt 1987). Cardinal directions are easier to analyze because the frame of reference is fixed in space. However, the solution reported here is directly applicable for treating reference frames once a qualitative operation to integrate frames of different orientation is devised.

This work is part of a larger effort to understand how we describe and reason about space and spatial situations. In particular, within the research initiative 2, 'Languages of Spatial Relations' of the National Center for Geographic Information and Analysis (NCGIA 1989) a need for multiple formal descriptions of spatial reasoning—both quantitative-analytical and qualitative—became evident (Mark, et al. 1989, Mark and Frank 1990, Frank and Mark 1991, Frank 1990, Frank 1991).

The structure of this paper is as follows: the next section discusses previous work. Then the two basic operations on direction symbols and their properties are formalized, using geometric intuition; we also define what 'exact qualitative spatial reasoning' means. Two examples of systems of cardinal directions are constructed and then compared. The paper concludes with some research questions for future work.

2. Related work

A standard approach to spatial reasoning is to translate the problem posed into analytical geometry and to use quantitative methods for its solution. Many problems can be conveniently reformulated as optimization with a set of constraints, e.g. location of a resource or shortest path question. A similar approach has been applied to understand spatial references in natural language text (Herskovits 1986, Nirenburg and Raskin 1987, Retz-Schmidt 1987). A special problem is posed by the inherent uncertainties in these descriptions and their translation into an analytical format. (McDermott and Davis 1984) introduced a method using 'fuzz' and in (Dutta 1988, Dutta 1990, Retz-Schmidt 1987) fuzzy logic (Zadeh 1974) is used to combine such approximately metric data.

An entirely qualitative approach was utilized in the work on symbolic projections (Chang, et al. 1990). They translate exact metric information, primarily about objects in pictures, into a qualitative form Segments in pictures are projected vertically and horizontally, and their order of appearance is encoded in two strings. Spatial reasoning, especially spatial queries, are executed as fast substring searches (Chang, et al. 1987).

One is tempted to apply first-order predicate calculus to spatial reasoning with directions:

> "The direction relation NORTH. From the transitive property of NORTH one can conclude that if A is NORTH of B and B is NORTH of C then A must be NORTH of C as well (Mark, et al. 1989)"

This leads to complex rules of inference, stating conditions for points and the directional relations between them. Hernández combines directional and topological relations and gives rules for the deduction of spatial relations based on multiple, perspective observations (Hernández 1990).

Our concern is different from (Peuquet and Zhan 1987), where 'an algorithm to determine the directional relationship between arbitrarily-shaped polygons in the plane' is given and no inferences from given directional relations are drawn.

3. An algebra of cardinal directions

The intuitive properties of cardinal directions are described in the form of an algebra with two operations applicable to direction symbols:

- the reversing of the direction of travel (*inverse*), and
- the combination of the direction symbol of two consecutive segments of a path (*combination*).

The operational meaning of cardinal direction is captured in a set of formal axioms. These axioms define the properties of cardinal directions. The rules are, for a part, formally similar to vector algebra. We find that the axioms deduced from geometric intuition are approximating geometrically exact reasoning and as a system logically contradictory.

3.1. Cardinal Directions

Direction is a binary function from two points in the plane (P_1, P_2) that map onto a symbolic direction d. The specific directional symbols available depend on the system of directions used, e.g., d_4 = {N, E, S, W} or more extensive d_8 = {N, NE, E, SE, S, SW, W, NW}.

We avoid the limitation, that direction is only meaningful if the two points are different and introduce a special symbol 0 (for 'zero'), in algebra usually called identity. It means 'two points too close for a direction to be determined'. This simplifies the rules and increases deductive power.

3.2. Reversing direction

Cardinal directions depend on the direction of travel. If a direction is given for a line segment between points P_1 and P_2, the direction from P_2 to P_1 can be deduced (Peuquet and Zhan 1987, Freeman 1975). This operation is called *inverse* (Figure 1), with

inv (dir (P_1, P_2)) = dir (P_2, P_1) and inv (inv (d)) = d.

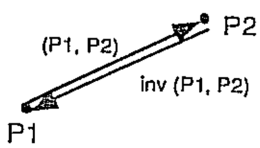

Figure 1: Inverse

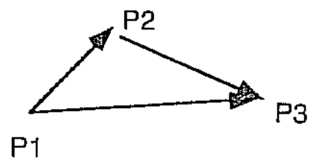

Figure 2: Combination

3.3. Combination

The second operation combines the directions of two contiguous line segments, such that the end point of the first direction is the start point of the second one (Figure 2).

$d_1 \infty d_2 = d_3$ means dir (P_1,P_2) ∞ dir (P_2, P_3) = dir (P_1, P_3)

It is not necessary that combination is commutative ($a \infty b \neq b \infty a$), but this is the case for both examples of definitions for cardinal directions studied here, are.

Associativity: Combinations of more than two directions should be independent of the order in which they are combined:

$a \infty (b \infty c) = (a \infty b) \infty c = a \infty b \infty c$ (associative law)

This rule follows immediately from a figure or from the definition of combination in terms of line segments.

Identity: Adding the direction from a point to itself, dir(P_1,P_1) = 0, to any other direction does not change the direction.

Algebraic definition of inverse: In algebra, an inverse to a binary operation is defined such that a value, combined with its inverse, results in the identity value. Geometrically the inverse is the line segment that combines with a given other line segment to lead back to the start (see Figure 3).

inv (d) ∞ d = 0 and d ∞ inv (d) = 0

Computing the combination of two directions, where one is the inverse of the other, is in the general case an *approximation* and not Euclidean exact. The degree of error depends on the definition of 0 used and the difference in the distance between the points - if they are the same, the inference rule is exact. This represents a type of reasoning like: New York is east of San Francisco, San Francisco is west of Baltimore, thus New York is too close to Baltimore (in the frame of reference of the continent) to determine the direction. In general one observes that deduction based on directions alone is only applicable if all the distances are of the same order of magnitude.

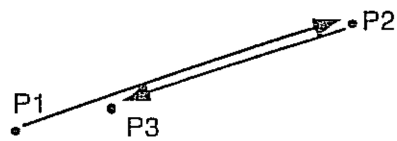

Figure 3: $d \infty \text{inv}(d) = 0$

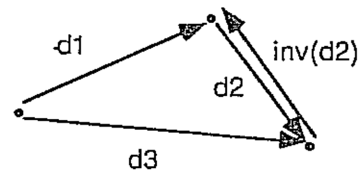

Figure 4: $d_1 = d_3 \infty \text{inv}(d_2)$

The inverse is also used to compute the completion of one path to another (Figure 4) and one finds that a combined path is piece-wise reversible $\text{inv}(a \infty b) = \text{inv}(b) \infty \text{inv}(a)$.

Idempotent: If one combines two line segments with the same direction, one expects that the result maintains the same direction. This is transitivity for a direction relation:

from d (A,B) and d (B,C) follows d (A,C)

This rule for directional symbols is substantially different from the corresponding rule for vector addition. From an algebraic point of view, all directions are idempotent:

$d \infty d = d$, for any d.

3.4. Definition of Euclidean exact reasoning

The rules for qualitative spatial reasoning given are compared with the quantitative methods using analytical geometry. The following definition gives a precise framework for such a comparison. A qualitative rule is called *Euclidean exact,* if the result of applying the rule is the same as if we had translated the data to analytical geometry and applied the equivalent functions, i.e. if we have a homomorphism. Otherwise it is called *Euclidean approximate.*

$$\text{dir}(P_1, P_2) \infty \text{dir}(P_2, P_3) = \text{dir}((P_1, P_2) + (P_2, P_3))$$

3.5. Summary of Properties of Cardinal Directions

The basic rules for cardinal directions and the operations of inverse and combination are:

- The combination operation is associative (1').
- The direction between a point and itself is a special symbol 0, called *identity* (1) (2')
- The direction between a point and another is the *inverse* of the direction between the other point and the first (2) (3').
- The set of directions is closed under inverse and combination.
- Combining two equal directions results in the same direction (*idempotent*, transitivity for direction relation) (3) - an approximate rule
- The combination can be inverted (4).
- Combination is piece-wise invertible (5).

Properties of direction		Group properties	
$\text{dir}(P_1, P_1) = 0$	(1)	$d \infty (d \infty d) = (d \infty d) \infty d$	(1')
$\text{dir}(P_1, P_2) = \text{inv}(\text{dir}(P_2, P_1))$	(2)	$d \infty 0 = 0 \infty d = d$	(2')
$d \infty d = d$	(3)	$d \infty \text{inv}(d) = 0$	(3')
for any a, b in D exist unique x in D such that $a \infty x = b$ and $x \infty a = b$	(4)		
$\text{inv}(a \infty b) = \text{inv}(a) \infty \text{inv}(b)$	(5)		

Several of the properties of directions are similar to properties of algebraic groups or follow immediately from them. Unfortunately, the approximate cancellation rule (3') d ∞ inv (d) = 0 leads to a contradiction with the remaining postulates. Searching for an inverse x for any d ∞ x = 0, we find x = inv (d) (using (3)) or x = 0 (using 3'). A standard solution is to define a non approximate rule, d ∞ inv (d) = {d, 0, inv (d)}, which reinstates logical consistency. We prefer the plausible reasoning solution and use (3') but give up associativity (1') and uniqueness of the inverse (4).

4. Two Examples of Systems of Cardinal Directions

Two examples of systems of cardinal directions are studied, both using the same set of eight directional symbols (plus the identity). One is based on cone-shaped (or triangular) areas of acceptance, the other is based on projections. These two semantics for cardinal direction can be related to Jackendoff's principles of centrality, necessity and typicality (Jackendoff 1983) as pointed out by Peuquet (Mark, et al. 1989, p. 24).

4.1. Directions in 8 or More Cones

Cardinal directions relate the angular direction between a position and a destination to some directions fixed in space. An angular direction is assigned the nearest named direction which results in cone shaped areas for which a symbolic direction is applicable. This model has the property that "the area of acceptance for any given direction increases with distance" (Peuquet and Zhan 1987, p. 66) (with additional references) and is sometimes called 'triangular'.

We use the set of directional symbols

V_9 = {N, NE, E, SE, S, SW, W, NW, 0}.

and define a turn of an eighth anti-clockwise:

e (N) = NE, e (NE) = E, e (E) = SE, ... , e (NW) = N,   e (0) = 0

with 8 eighth turns being the identity function e^8 (d) = d. The inverse is defined as 4 eighth turns inv (d) = e^4 (d). The rules for combination of directions are(2') (3) and the approximative rule (3'). This allows for a deduction of about one third of possible combinations. A set of averaging rules, which allow the combination of directional values which are apart by 1 or 2 eighth turns, is necessary to complete the system. For example N is combined with NE to yield N.

e (e (d)) ∞ d = e (d),   e (d) ∞ d = d, e (d) ∞ inv (d) = 0, d ∞ e (d) = d, etc.

In this system, from all the 81 pairs of values (64 for the subset without 0) combinations can be inferred, but most of them only approximately. Written as a table, where lower case denotes Euclidean approximate inferences:

	N	NE	E	SE	S	SW	W	NW	0
N	N	n	ne	o	o	o	nw	nw	N
NE	n	NE	ne	e	o	o	o	n	NE
E	ne	ne	E	e	se	o	o	o	E
SE	o	e	e	SE	se	s	o	o	SE
S	o	o	se	se	S	s	sw	o	S
SW	o	o	o	s	s	SW	sw	w	SW
W	nw	o	o	o	sw	sw	W	w	W
NW	nw	n	o	o	o	w	w	NW	NW
0	N	NE	E	SE	S	SW	W	NW	0

Directions so defined do not fulfill all the requirements, because they violate the associative property (i.e. 328 out of 729 cases); for example $(S \infty N) \infty E = 0 \infty E = E$ but

$S \infty (N \infty E) = S \infty NE = 0$. The evaluation of a complex expression depends on the order. The differences are minor and are probably a reflection of the inherent vagueness of the concept of directions (Herskovits 1986, p. 192)

4.2. Cardinal Directions Defined by Projections

4.2.1. Directions in 4 half-planes

One can define four directions such that they are pair-wise opposites (Peuquet and Zhan 1987) and each pair divides the plane into two half-planes. The direction operation assigns for each pair of points a combination of two directions, e.g., South and East for a total of 4 different directions (Figs. 5 and 6).

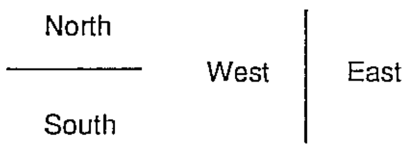

Figure 5: Two sets of half-planes.

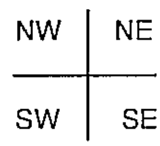

Figure 6: Directions defined by half-planes.

Another justification for this definition of cardinal direction is found in the structure that geographic longitude and latitude impose on the globe. Cone directions better represent the direction of 'going toward', whereas the 'half-planes' (or equivalent parts of the globe) better represent the relative position of points on the earth. Frequently, the two coincide.

In this system, the two projections can be dealt with individually. When the two projections in N-S and E-W are combined to form a single system with the directional symbols:

$V_4 = \{ NE, NW, SE, SW \}$

only the trivial cases ($NE \infty NE = NE$, etc.) can be resolved.

4.2.2. Projection-Based Directions with Neutral Zone

If points which are near to due north (or west, east, south) are not assigned a second direction, i.e. one does not decide whether or not such a point is more east or west, one divides effectively the plane into 9 regions (Figure 7): a central neutral area, four regions where only one direction applies and 4 regions where two are used. We define for N-S three values for direction d_{ns} $\{N, P, S\}$ and for the E - W direction the values d_{ew} $\{E, Q, W\}$.

NW	N	NE
W	O_C	E
SW	S	SE

Figure 7: Directions with neutral zone

It is important to note that the width of the 'neutral zone' is not defined a priori. Its size is effectively decided when the directional values are assigned and a decision is made that P_2 is north (not north-west or north-east) of P_1.The algebra deals only with the directional symbols, not how they are assigned. The system assumes that these decisions are consistently made in order to determine if a deduction rule is Euclidean exact or not.

Allowing a neutral zone introduces a 'tolerance geometry' point of view. Whenever an identity direction dir $(P_1, P_2) = 0$ is assigned in cases where $P_1 \neq P_2$, the transitivity assumption of equality is violated (Robert 1973, Zeeman 1962).

In one projection, selecting $d_{ns}= \{N, P, S\}$ as the prototype, the inverse operation is defined as inv (N) = S, inv (S)= N, inv (P) = P. For the combination, we require the rules (3), (2') and (3'). These rules together violate associativity. In 200 of the 729 cases, the result depends on the order of evaluation.

The two projections in N-S and E-W are then combined to form a single system, in which for each line segment one of 9 combinations of directions are assigned.

$V9 = \{NE, NQ, NW, PE, PW, PQ, SE, SQ, SW\}$

Adding some syntactic sugar, P and Q are eliminated (replacing PQ by 0)

$V9 = \{NE, N, NW, E, W, 0, SE, S, SW\}$

If any of the results of the two projections is approximate, the total result is considered approximate reasoning.

The inverse operation is combined from the inverse for each projection, written as a table:

d	NE	N	NW	E	W	0	SE	S	SW
inv(d) =	SW	S	SE	W	E	0	NW	N	NE

The combination operation is defined as the combination for each projection. Using the three rules, we can compute the values for each combination. Written as a table (again, lower case indicates approximate reasoning):

$*_d$	N	NE	E	SE	S	SW	W	NW	0
N	N	NE	NE	e	o	w	NW	NW	N
NE	NE	NE	NE	e	e	o	n	n	NE
E	NE	NE	E	SE	SE	s	o	n	E
SE	e	e	SE	SE	SE	s	s	o	SE
S	o	e	SE	SE	S	SW	SW	w	S
SW	w	o	s	s	SW	SW	SW	w	SW
W	NW	n	o	s	SW	SW	W	NW	W
NW	NW	n	n	o	w	w	NW	NW	NW
0	N	NE	E	SE	S	SW	W	NW	0

4.3. Assessment

The power of the 8 direction cone-shaped and the 4 half-plane based directional system are similar. Each system uses 9 directional symbols, 8 cone directions plus identity on one hand, the Cartesian product of 3 values (2 directional symbols and 1 identity symbol) for each projection on the other hand. The same three rules are used to build both systems, but the cone-shaped system had to be completed with a number of 'averaging rules'

Introducing the identity symbol 0 increases the number of deduction in both cases considerably. Only 8 out of 64 combinations could be resolved for cone shaped directions and only trivial cases can be resolved for the half-plane based system. Without the identity symbol the systems with identity allow conclusions for any pair of input values (81 different pairs) at least approximately. The comparison reveals that the cone-shaped directions result in more cases of approximative results than the projection-based system (56 vs. 32) and that it yields more often the value 0 (25 vs. 9). Considering the actual values (other than 0) deduced, we see differences only for results where the 'averaging rules' are used for the cone-shaped directions, and the values differ by one eighth turn only. The two systems produce essentially the same deduction results.

Both systems violate some of the desired properties. One can easily observe that associativity is not guaranteed, but the differences seem not to be very significant.

An implementation of these rules and a comparison of the computed combinations with the exact value was carried out and these results confirm the theoretical findings. Comparing all possible 10^6 combinations in a grid of 10 by 10 points (with a neutral zone of 3 for the projection-based directions) shows that the results for the projection-based directions are correct in 50% of cases and for cone-shaped directions in only 25%. The result 0 is the outcome of 18% of all cases for the projection-based, but 61% for the cone-shaped directions. The direction-based system with an extended neutral zone produces in 2% of all cases, a result that is a quarter turn off. Otherwise, the deviation from the correct result is never more than one eighth of a turn, namely in 13% of all cases for cone-shaped and 26% for projection-based direction systems. In summary, the projection-based system of directions produces in 80% of all cases a result that is within 45° and otherwise the value 0.

5. Conclusions

This paper introduced a system for inference rules for qualitative spatial reasoning with cardinal distances from an algebraic point of view. Two operations, *inverse* and *combination* are applied to direction symbols and their meanings formalized with a set of axioms. Three requirements, that such directions should fulfill, are (1) the direction from a point to itself is a special value, meaning 'too close to determine a direction', (2) every direction has an inverse, namely the direction from the end point to the start point of the line segment, and (3) the combination of two lines segments with the same direction results in a line segment with the same direction.

These rules can be used to identify some systems which are not suitable for cardinal directions, e.g., a system with an uneven number of cardinals. In order to compare the qualitative rules with quantitative calculations, we define the notion of 'Euclidean exact', using a homomorphism. A deduction rule is called 'Euclidean exact' if it produces the same results as those obtained using Euclidean geometry operations.

Two different systems for cardinal directions were discussed in detail, both fulfilling the requirements for directions. One is based on cone-shaped (or triangular) directions, the other deals with directions in two orthogonal projections. The introduction of the identity element 0 simplifies the reasoning rules in both cases and increases the power for both, cone and projection-based directional systems. The half-plane based system of directions is slightly easier to describe and formalize, as we can deal with each projection separately, yielding simpler inference rules.

Both systems yield results for all the 81 different inputs for the combination operation, but the projection-based system does more often yield an Euclidean exact result than the cone-based one (49 vs. 25 cases). It also produces less often the value 0 (9 vs. 25 cases). Another important result is that the two systems do not differ substantially in their conclusions, if definite conclusions can be drawn, i.e. not the value 0. This reduces the potential for testing with human subjects to find out which system they use, observing cases where the conclusion using one or the other line of reasoning would yield different results.

We have implemented these deduction rules and compared the results obtained for all combinations in a regular grid. The projection-based system results in 53% of all cases in exact results and in another 26% in results which are not more than 45° off. In 18% of all cases the application of the rules yields a value of 0. The results for the cone-shaped directions are less accurate. The methods shown here can be used to quickly assess whether the combination of two directions yields a value that falls within some limits, and a more accurate and slower computation should be done.

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