Formalization of Families of Categorical Coverages[1]

ANDREW U. FRANK[2],
GARY S. VOLTA,
and MATT McGRANAGHAN[3]

National Center for Geographic Information and Analysis and
Department of Surveying Engineering, 337 Boardman Hall,
University of Maine, Orono, ME 04469, U.S.A.

Abstract. Categorical coverages are popular for the presentation of thematic data in cartographic form and have gained widespread use. Routines for their preparation are included in most GIS software. Categorical coverages are simple to implement and easy to understand. The popularity of the categorical coverage is closely related to the intuitive rules that determine its behavior. This paper presents a formalization of these rules based on the concepts of a partition of space and the refinement of such partitions of space.

A family of categorical coverages is defined as all categorical coverages that can be produced from a data set through aggregations of categories. It is shown that categorical coverages are partially ordered by 'refinement' of partitions. This ordering is preserved in the mapping from the ordering of the partition of the attribute domain to the partition of space. This formalization is not only useful for implementors of GIS, but the list of possible operations and their properties is the base for the design of a user interface to produce and manipulate categorical coverages.

The thematic definition of categorical coverages is directly related to the cartographic rendering. Changes in the thematic definition, i.e. changes of the map legend, can be automatically propagated to the rendering. Limitation of cartographic rendering, e.g. resolution of the screen, limits the presentation of the result. These influences from the rendering cannot automatically be translated into changes in the subdivisions of the categories. This may explain why good graphical presentation is still an art and the production of thematic maps is not fully automated.

1. Introduction

"Thematic maps concentrate on the spatial variations of the form of a single attribute or relationship" (Robinson *et al*. 1984, p.13). The theme or set of attributes in a thematic map is often shown in a way that similar attribute values are

[1] Funding from NSF for the NCGIA under grant SES 88-10917 and from Intergraph Corporation is gratefully acknowledged.

[2] Current address: Department of Geo Information (E127), Technical University Vienna, Gusshausstrasse 27-29, A-1040 Vienna Austria.

[3] Current address: Geography Department, University of Hawai, 2424 Maile Way, HI 96822.

grouped into areal units. When these polygons are mutually exclusive and collectively exhaustive over the area of interest, the map is called a categorical coverage (Beard 1988).

Categorical coverages are a very important type of thematic map related to the classical choropleth map, which presents spatially arranged attribute information for delimited areas on the ground (Robinson *et al*. 1984). They are often used to map statistical data for enumeration districts. The categorical coverage has been extensively used in GIS since implementation in various software products such as SYMAP and GEOGRAPH of the Laboratory for Computer Graphics and Spatial Analysis at Harvard University (Schmidt and Zafft 1975).

A categorical coverage can easily be transformed into another more or less detailed coverage by merging or splitting attribute categories. A family of categorical coverages is defined as all possible categorical coverages that can be deduced from a single, most detailed coverage. Each member of the family will preserve a different amount of the original categorical detail. In an analysis, a user selects the most appropriate level of aggregation for the task at hand.

Categorical coverages are distributed by the value-added resellers, who combine standard boundary files with socio-economic data from census and similar organizations. A buyer receives not only the most detailed coverage, but the full family of coverages, formed by all possible aggregations which can be deduced from this data set. If a cartographer user (Morrison 1994) uses this data set, it is important to provide him with easily understandable methods to control the rendering in order to make it communicate what he is interested in.

Categorical coverages are intuitively understandable and are easy to implement. They are a general case of multi-scale descriptions, where data at different levels of resolution (or generalization) are represented (Bertolotto, De Floriani, and Marzano 1995) . In this paper, we formalize the concept of the family of categorical coverages and its properties, based on the mathematical notions of partitions of space and refinements of them. A categorical coverage is a partition of space induced by a partition of the attribute domain into categories. Partitions can be ordered by refinement and we find that if one partition of the attribute domain is a refinement of another, the induced spatial partition is a refinement of the partition in the spatial domain as well. Therefore, the creation of the induced spatial partition from a partition of the attribute domain is an order-preserving mapping.

In contrast, reorganization of the spatial partition by aggregation of adjoining areas does, in general, not lead to an aggregation in the map legend. Changes to the partition of the attribute domain, i.e. changes in the map legend, can be automatically propagated to the spatial partition. The reverse is not true: changes in the spatial domain do not propagate to the thematic domain. All operations on a categorical coverage should be applied to the partition of the attribute domain. The standard operation of merging areas too small to be mapped clearly with their adjoiners will lead to an approximation.

Such a formalization is useful for implementing a GIS because a clean theory simplifies the software construction enormously (Kuhn 1993) . This leads to a crisp user interface because the formalization defines what concepts and operations must be visible to the user. These concepts and operations are separable from the artifacts of a specific implementation. If a consistent, compact theory can be found, a conceptual standardization of the user interface is achieved. This is analogous to the beneficial normalization of the database management interface (Codd 1982) and in raster GIS, with Tomlin's map algebra (Tomlin 1983, 1989) or the definition of topological relations through the nine-intersections (Egenhofer and Franzosa 1991) .

The result contributes to a theory to explain "how maps work" (MacEachren 1995) . It explains the rules of this particular spatial representation and its limitation. This discussion concentrates on the logical aspects of manipulating partitions of geographic and attribute space and does not address the cartographic display issues in representing the areal units, the lines bounding them, or line generalization (for an extensive treatment, see (Beard 1988)). In consequence, scale, size of minimal mapped area, appropriate levels of thematic and spatial resolution, and so on, are not treated, though they must be considered to build complete systems.

The work presented here is part of a larger effort to understand how space and spatial situations are described and interacted with by users. It is related to and benefits from results from research initiatives of the National Center for Geographic Information and Analysis (NCGIA) I-2, 'Languages of Spatial Relations' (Mark *et al.* 1989) and also I-3, 'Multiple Representations' (Buttenfield 1989, Buttenfield and McMaster 1991). It is a direct contribution to the NCGIA Research Initiative 13 'User interfaces for GIS' (Mark *et al.* 1992).

This paper first covers the definitions and formalizes basic concepts to discuss categorical coverages. In section 4 the spatial partition induced by a partition of the theme is explained; in the following section the appropriate operations are defined and a design of a user interface presented. Section 6 covers cartographic considerations to improve the rendering of a categorical coverage.

2. Definition and Examples of Categorical Coverages

A categorical coverage consists of two connected components: (1) a spatial component and (2) a thematic component. Using Sinton's terminology (Sinton 1978), categorical coverages hold time fixed, control for theme, and measure the location, by searching for the largest areas with uniform properties. (This is in contrast to choropleth maps, in which location is controlled and theme is varied). The prototypical example for a categorical coverage is often the soils map, which shows areas of uniform soil type distributed in space. A simple example of such a map is presented in Figure 1. The same methods can be used to map land use categories, population densities, and so on.

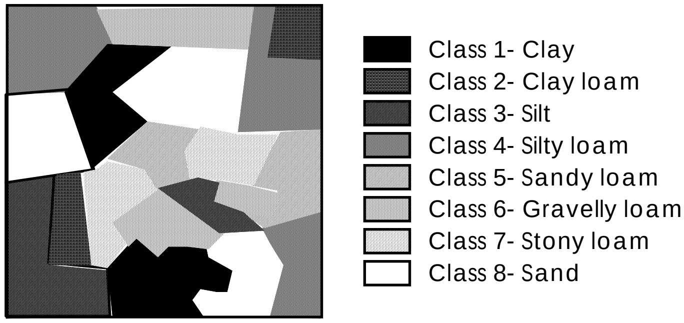


Figure 1. Simple soils map.

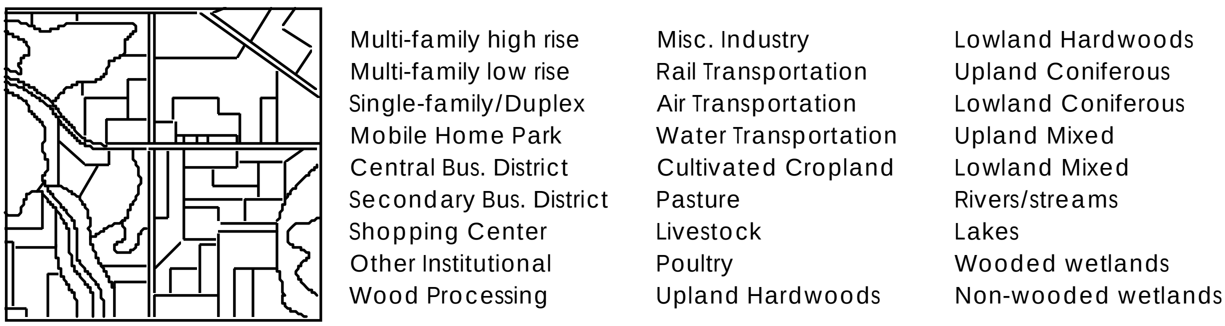


Figure 2. Land use/land cover map and list of categories (adapted from Anderson(1976)).

2.1 *Definitions of a Categorical Coverage*

Geographic information that represents the distribution of categories has been called *area-class data* (Bunge 1966). This terminology of area-class data was adopted by Mark and Csillag (1989) and defined as "for every location (x, y) in some region of the plane, there is either a class to which an observation made at that point would belong, or a probability that the observation falls into each class within some set of classes".

Chrisman and Beard have also studied categorical coverages and use the following definitions and terminology:

> "Chrisman (1982) defines a categorical coverage as: 'an exhaustive partitioning of a two dimensional space into arbitrarily shaped zones which are defined by membership in a particular category of a classification scheme' " (Beard 1988).
>
> "Categories and zones should not be confused. The categories are conceptual entities conceived by the human mind on examination of the landscape. The zones become the physical, spatial manifestations of the concept." (Beard 1988)

2.2 *Example of a Categorical Coverage*

The categorical coverage is often used to represent land use/land cover data. Figure 2 illustrates an example of a typical map and set of associated land use/land cover categories. In this example, every area shown in the categorical coverage on the left corresponds to one and only one of the attribute categories shown on the right.

This set of land use/land cover categories is a subset of the possible categories. Even in this small subset, similarities can be seen between categories and similar categories might be grouped into more general ones to reduce the categorical or spatial complexity to a level appropriate for a particular use.

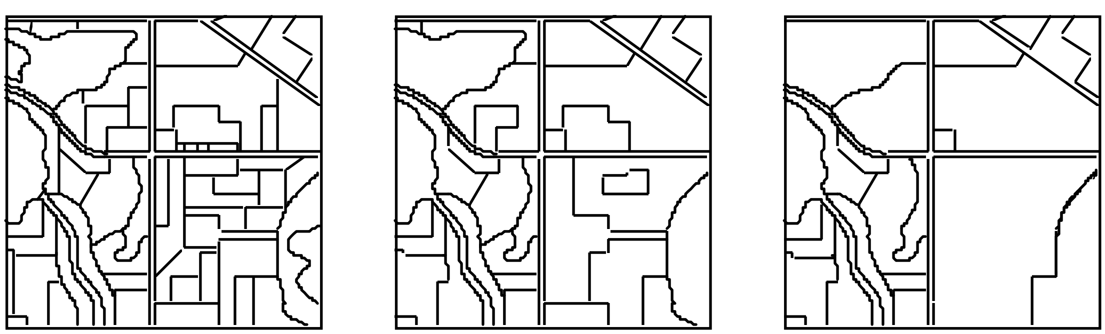
2.3 *Differences between Categorical Coverage and Choropleth Map*

For a categorical coverage, the primary step is the differentiation of the theme into a number of categories. Each category may consist of one value, several distinct values or intervals of attribute values for the theme or attributes of the coverage. The categories then determine the zones on the map, as each zone is constructed as the area where the attribute value of the theme falls uniformly in one of the categories ("assumption of homogeneous zones" (Beard 1988)).

For a choropleth map, which geometrically appears to be similar to a categorical coverage, the zones are predetermined mapping units, for which the best (e.g. average) value for theme is determined and depicted. The determination of the zone is therefore the primary step, the mapping (and sometimes categorization) of the representative theme attribute value is secondary.

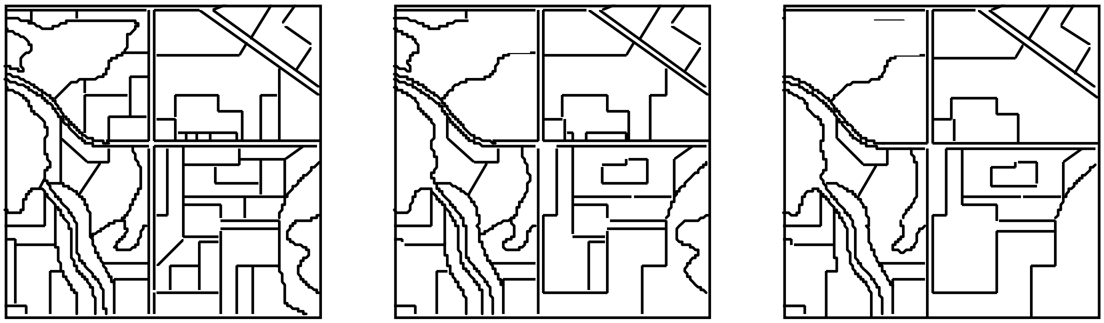
2.4 *Definition of a Family of Categorical Coverages*

From an initial categorical coverage, other categorical coverages on the same theme can be created by aggregating categories. This may induce an aggregation of zones. Further aggregations of categories can be constructed, potentially yielding a new spatial representation of zones each time. In the following four figures, two such aggregation schemes are shown. Figure 3a and 4a are identical and correspond to the attribute values shown in Figure 2. Figure 5 shows the set of attribute values in Figure 2 and two successive category aggregations. The first set of categories is depicted by italic type and the corresponding zones are shown in Figure 3b. The second set of categories is depicted by bold type and the resulting zones are seen in Figure 3c. The same process with a different aggregation scheme is shown in Figures 4 and 6 respectively.



a. b. c.

Figure 3. Spatial representations of category groupings shown in Figure 4.



a b. c.

Figure 4. Spatial representations of category groupings shown in Figure 5.

1. Urban and Built Up

1.1 Residential
- 1.1.1 Multi-family high rise
- 1.1.2 Multi-family low rise
- 1.1.3 Single-family/Duplex
- 1.1.4 Mobile Home Park

1.2 Commercial
- 1.2.1 Central Bus. District
- 1.2.2 Secondary Bus. District
- 1.2.3 Shopping Center
- 1.2.4 Other Commercial

1.3 Industrial
- 1.3.1 Wood Processing
- 1.3.2 Misc. Industry

1.4 Transportation
- 1.4.1 Rail Transportation
- 1.4.2 Air Transportation
- 1.4.3 Water Transportation

2. Agricultural Land

2.1 Cropland/Pasture
- 2.1.1 Cultivated Cropland
- 2.1.2 Pasture

2.2 Confined Feeding
- 2.2.1 Livestock
- 2.2.2 Poultry

3. Water/Wetland

3.1 Lakes/waterways
- 3.1.1 Rivers/Streams
- 3.1.2 Lakes

3.2 Other Wetlands
- 3.2.1 Wooded Wetlands
- 3.2.2 Non-wooded wetlands

4. Forest

4.1 Deciduous
- 4.1.1 Upland Hardwoods
- 4.1.2 Upland Coniferous

4.2 Coniferous
- 4.2.1 Upland Mixed
- 4.2.2 Lowland Hardwoods

4.3 Mixed Forest
- 4.3.1 Lowland Coniferous
- 4.3.2 Lowland Mixed

Figure 5. Attribute groupings from Anderson(1976) corresponding to Figures 3a, 3b and 3c.

1. Residential
- *1.1 Multi-family*
  - 1.1.1 Multi-family high rise
  - 1.1.2 Multi-family low rise
- *1.2 Single family*
  - 1.2.1 Single-family/Duplex
  - 1.2.2 Mobile Home Park

2. Commercial
- *2.1 Districts*
  - 2.1.1 Central Bus. District
  - 2.1.2 Secondary Bus. District
- *2.2 Independent*
  - 2.2.1 Shopping Center
  - 2.2.2 Other Commercial

3. Industrial
- *3.1 Wood Processing*
- *3.2 Misc. Industry*
- *3.3 Rail Transportation*
- *3.4 Air Transportation*
- *3.5 Water Transportation*

4. Forest
- *4.1 Upland Forest*
  - 4.1.1 Upland Hardwoods
  - 4.1.2 Upland Coniferous
  - 4.1.3 Upland Mixed
- *4.2 Lowland Forest*
  - 4.2.1 Lowland Hardwoods
  - 4.2.2 Lowland Coniferous
  - 4.2.3 Lowland Mixed

5. Agricultural Land
- *5.1 Cultivated Cropland*
- *5.2 Pasture*
- *5.3 Livestock*
- *5.4 Poultry*

6. Water/Wetland
- *6.1 Lakes/waterways*
  - 6.1.1 Rivers/Streams
  - 6.1.2 Lakes
- *6.2 Other Wetlands*
  - 6.2.1 Wooded Wetlands
  - 6.2.2 Non-wooded wetlands

Figure 6. Alternative attribute groupings corresponding to Figures 4a, 4b, and 4c.

This process of aggregating attribute values into categories and the induced aggregation of zones results in what we define as a *family of categorical coverages*. The set of spatial representations shown in Figure 3 and 4 and the categories shown in Figure 5 and 6 are part of a family of categorical coverages as derived from the initial domain(s) of Figure 2.

3. Basic Concepts

In this section the concepts of a family of categorical coverages are formally defined and the relevant background from mathematics is presented. More extensive discussions can be found in introductory texts on discrete structures (Gill 1976, Preparata and Yeh 1973, (Piff 1991)). The terminology used for this discussion is adopted from Gill (1976).

3.1 *Partition*

A set is a collection of elements on which the regular operations intersection, union, complement and so on are defined. "A partition of a set A is a collection of subsets of A, so that each element in A belongs to exactly one of these subsets" (Preparata and Yeh 1973). The subsets of A are called the blocks of the partition. The rule that each element in A is in exactly one block dictates that the intersection of any two different blocks is empty and all the blocks together exhaust the set.

$p_i = \{ A_i \}$ is a partition of A if and only if

- A_i INTERSECT A_j = empty, for all i _ j

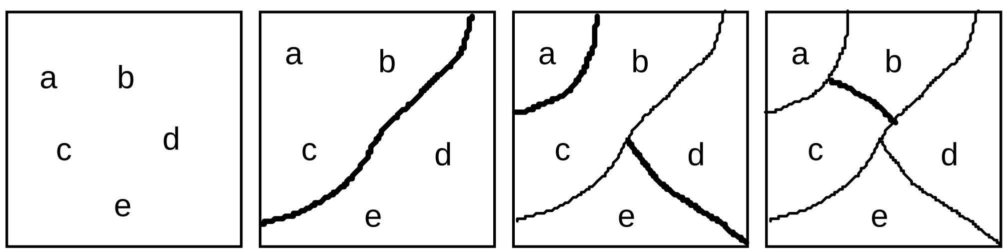
- UNION $A_i = A$

An equivalence relation on the set A induces a partition on A. A family of partitions is the collection of all partitions which are constructed from merging some of the blocks of a partition. A <u>complete family</u> of partitions is produced from the powerset of the N blocks of the original partition and contains therefore 2^N partitions. In general, a family of partitions will contain some subset of the partitions from the complete family.

A {a, b, c, d, e}
B {a, b, c}, {d, e}
C {a}, {b, c}, {d}, {e}
D {a}, {b}, {c}, {d}, {e}

Figure 7. Four partitions from the complete familiy of partitions.

Some of the implications of these definitions can be seen in Figure 7, which shows a family of partitions A, B, C, and D. D is composed of five blocks, each containing one element denoted by a lowercase letter in the set {a, b, c, d, e}. C is a partition with four blocks, one resulting by joining {b} and {c} into a single block {b, c}.



A. B. C. D.

Figure 8. Venn diagram of the refinement seen in Figure 7.

The partition B is a <u>refinement</u> of another partition A over the same set if every block B_i is a subset of some elements in a block A_j . The partition B can then be further refined to yield C (Figure 7). The Venn diagram (Figure 8) shows that for each refinement of this set, at least one additional boundary line (shown in bold) must be drawn. Mathematicians recognize that the subsets are either proper subsets (the refinement is then a proper refinement) or are equal to the previous subsets; thus a partition is always a (non-proper) refinement of itself.

Partitions are partially ordered by refinement. A <u>partial order</u> (²) is defined as a reflexive, antisymmetric, and transitive relation:

P_1 ² P_1 (1)

P_1 ² P_2 and P_2 ² P_1 => $P_1 = P_2$ (2)

P_1 ² P_2 and P_2 ² P_3 => P_1 ² P_3 (3)

A partial ordering on P is called a total ordering if furthermore:

for all P_i , P_j elements of P : P_i ² P_j or P_j ² P_i (4)

Refinement need not be a total order since it is possible that two partitions in the same family do not satisfy (4) for refinement. For example, the partitions shown in Figure 3b and Figure 4b cannot be derived from one another by refinement; however, the example in Figure 8 is totally ordered.

3.2 *Attribute Domain*

The theme of the categorical coverage is a domain (or set) of attribute values D= {d_1 ...d_n }, one of which describes the properties at a given location. Sets can be finite or infinite and partitions can be defined over both. The domain consists of a finite or infinite number of attribute values, which are at least discernible (nominal scale), but often have more internal structure. They may be ordered allowing subsets to be formed as intervals (Figure 9). This is the primary method to map an infinite domain of attribute values to a finite set. The set of values (intervals etc.) that are differentiated are often called the categories of the coverage, C={c_1 ...c_n }, (Beard 1988). The categories must form a partition of the domain of attribute values (i.e. categories are the blocks of the partition of the domain). This is typically achieved through a equivalence relation, which partitions the domain in a finite number of equivalence classes (Stevens 1946) .

Rainfall (in inches)

Attribute values D={0,1, 2, 3, 4, 5, 6, 7, 8, 9, 10 ...}

Categories of the coverage C_1 ={0-2, 3-4, 5-6, 7-8, 9-10, >10}

Categories of the coverage C_2 ={0-5, 6-10, >10}

Figure 9. Two interval categories derived from attribute domain.

3.3 *Space*

Categorical coverages cover a selected area G, a subset of space, thought of as a two-dimensional plane, possibly embedded in three-dimensional geographic space. This space is conceptualized as regular Euclidean space and typically mapped onto R^2 (i.e., provided with a coordinate system) . The zones of the categorical coverage form a partition of space. The zones, which are the blocks of the partition of space, are exhaustive of the subset of space and do not overlap (Beard, 1988).

3.4 *Theme Values*

For each point of the subset of space G, a value from the attribute domain D is determined (area-class data). The theme of a thematic map can be seen as a mapping from space to an attribute value. This is similar to the concept of 'geographic reality' introduced by Goodchild. "The fundamental element of geographical information is the tuple

$$T = \{x, y, t_1 \ldots t_n\},$$

giving the values of a set of spatial variables at the location (x,y)" (Goodchild 1992) .

4. Categorical Coverage as an Induced Partition of Space

A categorical coverage is a set of spatial zones constructed by taking a partition of the attribute domain (a set of categories) so that for all points in a zone the attribute value is in the same block of the partition of the attribute domain. The zones form equivalence classes in the spatial domain, where the theme value associated with each point is in the same equivalence class in the attribute domain.

Each partition of the attribute domain results in a different set of zones. Therefore, the set of categories induces the zones. This reflects the basic concept of a categorical coverage that one selects first the set of categories and then constructs the zones. In particular, any partition of the attribute domain leads to an induced partition of space.

To illustrate this notion, a mapping f exists for each point (x,y) in Q onto an attribute value d_i in the domain D of the theme, f(x,y) -> d_i in D. There is a partition of the attribute values d_i in D into a partition C of categories c_i (the blocks of the partition C) . This is also a mapping from the d_i onto the c_i , p(d_i) -> c_i . The composition of f and p is a mapping f'(x,y) -> c_i . Zones are formed from contiguous points in Q that have the same category value c_i . These zones are exhaustive because for every point (x,y) a value c_i is determined and their intersections are empty, because for every point (x,y) only a single value c_i results.

Categorical coverages only 'work' for attribute domains (themes) with considerable spatial auto correlation. If the value for every point (x,y) is different from the values of its neighbors, zones of single points result and a meaningful categorical coverage map cannot be drawn.

One observes that the refinement of the partitions of the attribute domain induces a refinement of the partition of space. For two categorical coverages of the same theme M_1 and M_2 and the partitions of the attribute domain C_1 and C_2 , which induced M_1 and M_2 , M_2 is a refinement of M_1 if and only if the partition of the attribute domain C_2 is a refinement of the partition of the attribute domain C_1 .

For example, assume that C_1 is a minimal refinement of C_2 , so that all the categories in C_1 and C_2 are the same except for c_{1_i} and c_{1_j} , which are merged to form c_{2_i} . Then all zones in M_1 are the same as in M_2 except for the zones in M_1 that have the values c_{1_i} and c_{1_j} . These zones in M_1 are combined to form zones of the value c_{2_i} in M_2 .

To illustrate this notion, we return to our land use/land cover example. The attribute values of ’Central Business District’ and ’Secondary Business District’ in M_1 are merged to form ’Business District’ in M_2. As shown in Figure 10, the zones in M_1 of the value of ’Central Business District’ and ’Secondary Business District’ merge to form the zones in M_2 corresponding to ’Business District’.

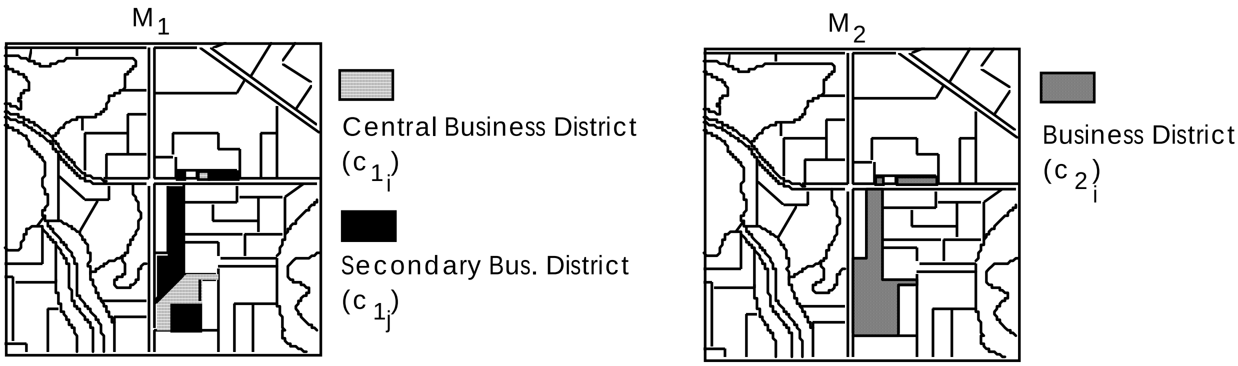
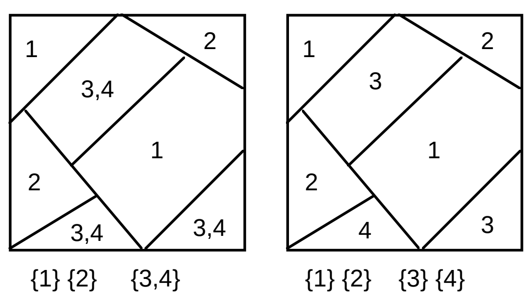


Figure 10. Zone behavior as a result of category refinement.

Refinement of partitions orders both the partitions of the attribute domain and the induced partitions of space in the same way. The construction of a categorical coverage preserves this ordering. Therefore it is possible to state that a categorical coverage is a refinement of another without detailing whether it is a refinement of the attribute domain or a refinement of the space partition (it is automatically a refinement of both). However, this is only true for refinement, not proper refinement: a partition of the attribute domain which is a proper refinement of another one only induces a refined partition of space (not a proper refinement of the partition of space). Figure 11 illustrates such a case. The geometric representation of the space is not affected by the refinement from 11a to 11b, although the thematic representation is affected. The three zones of the single category {3,4} in 11a are refined thematically to produce the two categories {3} and {4} in 11b without any geometric change.



a. b.

Figure 11. Refinement in attribute domain with no result in spatial domain.

A family of categorical coverages is a collection of categorical coverages using the same theme. Each consists of a partition of the attribute domain (the set of blocks) and the partition of space (the set of zones). A family of categorical coverages is partially ordered by 'refinement'. The complete family consists of all the different categorical coverages for this theme and forms a lattice (Gill 1976, Preparata and Yeh 1973)). Note that if the cardinality of the domain of attribute values of the theme is finite (infinite), then the cardinality of the complete family of categorical coverages is also finite (infinite).

For a finite domain of attribute values, a *most refined categorical coverage* exists. It is the one induced by the partition of the attribute domain, in which each attribute value forms its own category. In a GIS this most detailed categorical coverage is determined by the data stored and in principle all other coverages can be deduced from it. This notion was developed in (Peucker and Chrisman 1975) as the mutually exclusive and collectively exhaustive set of Least Common Geographic Unit (LCGU) that might result from the overlay of all the polygons for all of the attributes in a database. It is the basis for the relational model of GIS.

Conversely, there exists a *least refined categorical coverage* so that all attribute values contained within the same category form a single zone of the entire space. This type of categorical coverage is of little practical interest.

5. Operations on the Complete Family of Categorical Coverages

The complete family of categorical coverages contains all categorical coverages that can be constructed with a given theme. In order to select a categorical coverage map that communicates a desired message or allows one to understand some spatial phenomenon, a user needs appropriate tools. The ordering of the categorical coverages by refinement and the organization of the attribute domain are appropriate tools and should be used.

Categorical coverages form a formally definable object with a definite set of operations. It is therefore a prime candidate for a programmed component, which can be built into any GIS software. There is considerable industry interest (Udell 1994) in well defined, exchangeable components and several efforts to standardize such components in general, e.g., the COBRA standard, or for GIS in particular, e.g., the OGIS effort. Exchangeable software components provide not only savings in the production and maintenance of software, but perhaps even more important, reduce efforts in training of users because all systems work in this particular aspect in the same way.

5.1 *Refinement Operation*

For any given categorical coverage (which is not the most refined one) a more refined one can be constructed by splitting at least one category block into two (or more). Conversely, for any given categorical coverage (which is not the least refined one) a less refined one can be constructed by merging categories (a coarsening).

The user can navigate the lattice of partitions of the attribute domain (Figure 7), and for each of the partitions, the induced categorical coverage can be produced. In most practical circumstances the cardinality of the attribute domain is so large that the complete lattice of all partitions cannot be visualized directly and does not help during interaction. In the next subsection, general purpose tools are discussed and methods that exploit additional structure in the attribute domain are explored.

5.2 *Split and Merge Operators*

Starting with the given partition of the attribute domain, a refinement or coarsening step is directed by splitting or merging some of the current categories. Each category is a set of values from the attribute domain and the user can merge two categories to achieve a coarsening of the categorical coverage, or split a category into one or more smaller ones to achieve a refinement of the categorical coverage. These decisions are governed by the task and the semantics of the categories involved.

How these operations are implemented and presented to the user is not critical at the current level of discussion, although the interface should be determined such that the detail fits within the conventions of the particular GIS to maintain its internal consistency. Figure 12 sketches some of the necessary components in one possible design (for an alternative see (Volta and Egenhofer 1993)). It is based on direct manipulation of the set of categories and values associated with the categorical coverage. Values and categories can be selected from the scrolling list to which the appropriate operators can then be applied. A slightly different interface would be needed to handle interaction with predetermined or "existing" coverages and to handle the display output.

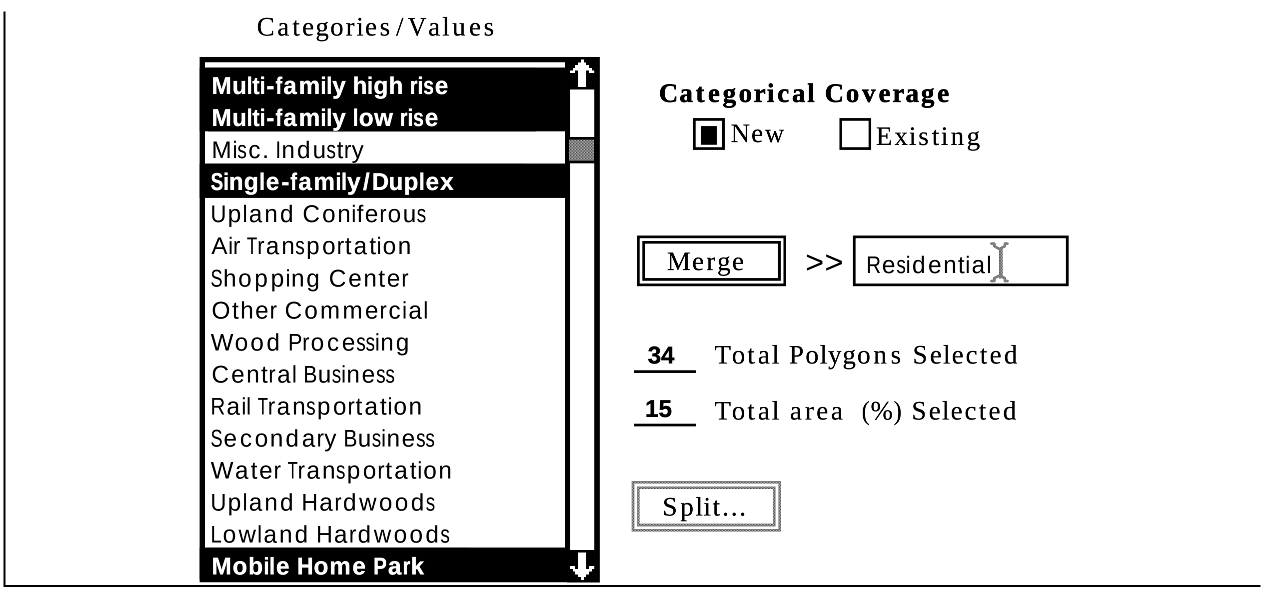


Figure 12. One possible implementation for category manipulation

5.3 *Structured Attribute Domains*

If the attribute domain has additional structure, for instance, if the attribute values are ordered, this structure must be used to aid the user in the selection and definition of the partition needed to complete the task. Ordered domains and hierarchical domains are two examples of structured domains.

5.3.1 Ordered domains

It is very common that the attribute values are ordered, and the categories are defined as intervals from this ordered set. A tool for the user can then be an operation to form a regular subdivision of the ordered domain in intervals by stating the number of categories desired. Another tool may show the attribute domain as a line and allow the user to click on this line to place category boundaries. Such tools are commonly found in many image processing and remote sensing systems (Ehlers 1990).

5.3.2 Hierarchical domains

Attribute values can often be ordered hierarchically, typical for taxonomies. For example, the categories from Figure 5 can be arranged in the taxonomy of Figure 13.

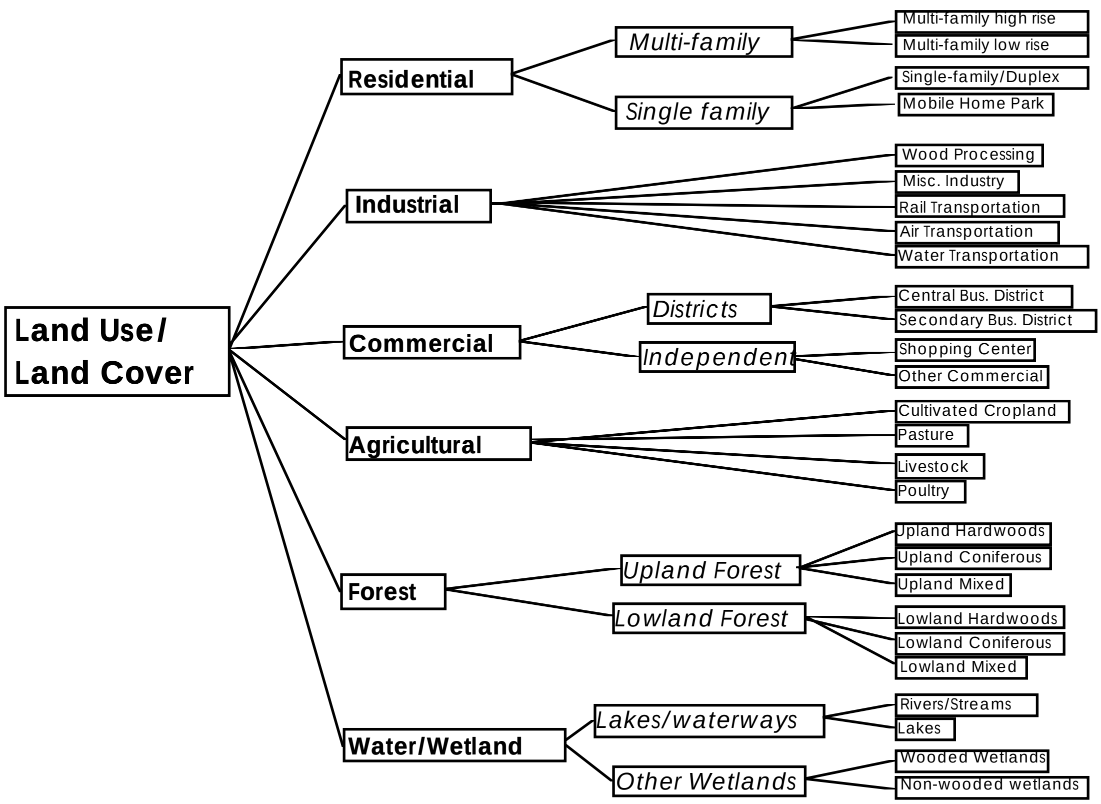


Figure 13. Hierarchical attribute domain taken from Figure 6.

For simple problems, the user can select a hierarchical level of detail and automatically get the corresponding categories and the categorical coverage. The user's task may be further complicated if the data are organized under multiple hierarchies rather than a single one. Appropriate methods are under investigation (Volta 1992).

6. Cartographic Rendering of Families of Categorical Coverages

In the previous sections the dependency of the spatial partition on the partition of the theme was stressed. This dependency is strictly functional and from a given partition of the theme a spatial subdivision follows immediately. If this spatial partition is rendered as a categorical coverage map, then the limitations of cartographic rendering influence the solution. A number of pragmatic rules define a feed-back from cartography to the partition of the theme, i.e. the map legend. These rules include

- total area covered by a block,
- minimal areas which can be mapped,
- number of different partitions which can be shown.

The feed-back can lead to changes in the thematic subdivision, but may also lead to changes in map scale, production method (e.g., black and white vs. color).

6.1 Total Area Covered by a Block

A thematic subdivision which covers only a minimal part of the area is most likely not important. It can be left out without the user losing information. The suppression improves often the cartographic appearance and thus communication. A pragmatic rule to resolve these cases is to merge blocks which cover only a small area with an appropriate, logically related thematic class.

6.2 Total Number of Blocks in a Partition

Each block in the thematic subdivision corresponds to a block in the spatial subdivision. They must be represented graphically with a pattern which can be differentiated from the patterns used to represent the other blocks. Guidelines for thematic map design recommend to use about 7 different themes (Imhof 1972) . A pragmatic solution is to merge the thematic blocks which include the smallest area with thematically related ones until only the desired number remains.

6.3 Minimal Cartographic Size

The area in a spatial subdivision representing one of the blocks in the partition of the theme, consists in general of multiple zones. If these zones are very small at the desired scale, they cannot be shown graphically. For each scale a minimal area is fixed and smaller areal objects cannot be effectively rendered. Other zones cannot be rendered because they do not have the required minimal size (e.g. thin and elongated objects). All these zones must be suppressed, which introduces a small error in the cartographic rendering. The different minimum conditions are fixed considering human visual abilities and the properties of printing press (e.g. a minimal area of 4 mm (Imhof 1972)) and the error remains small compared to the overall information in the map and other sources of error.

The rules used to adjust cartographic appearances are pragmatic, not strict. There are different pragmatic recommendations in the literature how generalization of zones should be done (Beard 1987). Arnberger gives 4 rules (Arnberger 1966) ; among them one to merge zones while preserving the total area for each category. They cannot be applied automatically, but must be user-controlled and used in accordance with the communication goal of the map (Beard 1987). Inappropriate usage can lead to loss of information in the map and deceive the map user. For example: Leaving out a small but important land use class may lead to the unjustified conclusion that this land use is not present in the area.

These operations contribute to the error in the map and reduce its quality although not necessarily to a noticeable degree. Nevertheless, it is important to separate the refinement operations discussed in section 5 and the reduction in the number of categories (subsections 6.1 and 6.2), which do not add error, from this operation, that potentially affects data quality.

It is therefore necessary to provide additional information to the user while manipulating the categories to enhance the available information for decision making. Information, such as the number of polygons within a particular category or group of categories, total area encompassed by each category (ground area or display area), average size (in ground or display units), could be used to achieve the optimal screen or map output. Other information such as adjoiner and neighborhood information follows from the topology and could be useful in the analysis of the coverage.

7. Conclusions

Categorical coverages are based on a mathematical structure which can be formalized in a direct way. It does not depend on the specifics of the properties of the attribute domain, but often the attribute domain comes with an additional structure, which could be utilized.

A categorical coverage consists of a partition of the attribute domain and the induced partition of geographic space. For each partition of the attribute domain into categories, an induced partition of space in non-overlapping, exhaustive zones exists. The partitions of the attribute domain are partially ordered by ’refinement’ and this order is translated to the induced spatial partition. It is thus meaningful to state that one categorical coverage is a refinement of another one (both a refinement in the attribute and in the spatial domain).

The major operations on a categorical coverage are related to the coarsening or refinement of the partition of the attribute domain. Simple tools to merge or split categories to achieve a coarser or more refined categorical coverage are the minimum a user needs. If the attribute domain has additional structure (e.g. the values are ordered or a hierarchical aggregation exists in the problem domain), the category manipulation tools should reflect and take advantage of this structure.

One must assume that the simplicity of the logical structure of categorical coverages has attributed to their widespread use. Understanding this structure aids the design of the user interface enormously by clarifying the operators that are necessary to manipulate categorical coverages. The similarity in the task structure of viewing a partition of categories and viewing a directory structure - even if not apparent at first consideration - allows to use tools developed and used at the ’general user’ level of an interface (e.g. the ’Explorer’ in Windows95(Stinson 1995)) capitalizing on the user’s knowledge of the interface.

Pragmatic rules provide feed-back from the cartographic rendering to the thematic subdivisions. They can be separated from the strict rules which translate the thematic subdivision into a spatial subdivision. These pragmatic rules are necessary to adapt the data to the limitations of the cartographic rendering and communication process. They are — like many other rules of map generalization — to be applied under user control to assure that the map communicates the intended message and the errors introduced are negligible.

Acknowledgments

The authors would like to thank Robert D. Franzosa and Max J. Egenhofer for their invaluable contributions to this paper. A spirited discussion with Peter Fisher considerably influenced the paper in a late stage and is gratefully acknowledged.

References

Arnberger, Erik. *Handbuch der thematischen Kartographie*. Vienna, Austria: 1966.

Anderson, J. R., Hardy, E. E., Roach, J. T. and Witmer, R. E., 1976, A land use and land cover classification system for use with remote sensor data. U.S.G.S. Report 964.

Bertolotto, M., De Floriani, L., and Marzano, P. “A Unifying Framework for Multilevel Description of Spatial Data.” In *Spatial Information Theory - A Theoretical Basis for GIS*, ed. Frank, A.U., and Kuhn, W. 259-278. 988. Berlin-Heidelberg-New York: Springer, 1995.

Beard, K., 1987, How to survive on a single detailed database. In: Proceedings of AUTO-CARTO 8, Eighth International Symposium on Computer-Assisted Cartography held in Baltimore, Maryland , Edited by N. R. Chrisman, pp. 211-220.

Beard, K. 1988. Multiple representations from a detailed database: a scheme for automated generalization, Ph.D. Thesis, University of Wisconsin - Madison.

Bunge, W., 1966, Theoretical Geography (Gleerup:Lund).

Buttenfield, B. P., 1989, Multiple representations: initiative 3 specialist meeting report. National Center for Geographic Information and Analysis (NCGIA) Report 89-3.

Buttenfield, B., and McMaster, R. B., 1991, Map Generalization: Making Rules for Knowledge Representation. (Essex: Longman Scientific & Technical).

Codd, E. F., 1982, Relational data base: a practical foundation for productivity. Communications of the ACM, 25, 109-117.

Egenhofer, Max J., and Franzosa, Robert D. “Point-set topological spatial relations.” *International Journal of Geographical Information Systems* 5 (2 1991): 161-174.

Ehlers, M., 1990, Image processing on the mac II: the DIRIGO system- a teaching experience, In: Proceedings of the ACSM/ASPRS Annual Convention, held in Denver, Colorado, pp. 113-120.

Gill, A.,1976, Applied Algebra for the Computer Sciences (Englewood Cliffs, NJ: Prentice-Hall).

Goodchild, Michael. “Geographical data modeling.” *Computers and Geosciences* 18 (4 1992): 401- 408.

Imhof, E. *Thematische Kartographie*. Berlin: 1972.

Kuhn, W. “Metaphors Create Theories for Users.” In *Spatial Information Theory: Theoretical Basis for GIS*, ed. Frank, A.U., and Campari, I. 366-376. 716. Heidelberg-Berlin: Springer Verlag, 1993.

MacEachren, Alan M. *How Maps Work - Representation, Visualization and Design*. New York: Guilford Press, 1995.

Mark, D., 1992, User interfaces: initiative 13 specialist meeting report. National Center for Geographic Information and Analysis (NCGIA) Report.

Mark, D., and Csillag, F., 1989, The nature of boundaries on 'area-class' maps. Cartographica, 26, 65-77.

Mark, D., Frank, A. U., Egenhofer, M. J., Freundschuh, S. M., McGranaghan, M. and White, R. M., 1989, Languages of spatial relations: initiative 2 specialist meeting report. National Center for Geographic Information and Analysis (NCGIA) Report 89-2.

Morrison, Joel. “The Paradigm shift in cartography: the use of electronic technology, digital spatial data and future needs.” In *Six International Symposium on Spatial Data Handling in Edinburgh*, edited by Waugh, T. C., and Healey, R. G., AGI, 1 - 28, 1994.

National Center for Geographic Information and Analysis (NCGIA), 1989, The research plan of the National Center for Geographic Information and Analysis. International Journal of Geographic Information Systems, 3, 117-136.

Peuker, T.K., and Chrisman, N. R., 1975, Geographic data structures, American Cartographer, 2, 55-69.

Piff, Mike. *Discrete Mathematics - An Introduction for Software Engineers*. Cambridge: Cambridge University Press, 1991.

Preparata, F.P., and Yeh, R.T., 1973, Introduction to Discrete Structures for Computer Science and Engineering (Reading, MA: Addison-Wesley).

Robinson, A. H., Sale, R. D., Morrison, J. L., and Muehrcke, P. C., 1984, Elements of Cartography 5th ed. (New York: John Wiley & Sons).

Schmidt, A. H., and Zafft, W. A., 1975, Programs of the Harvard University Laboratory for Computer Graphics and Spatial Analysis. In: Display and Analysis of Spatial Data. Edited by J. C. Davis and M. J. McCullagh. 231-243. (London: John Wiley and Sons).

Sinton. 1978. The inherent structure of information as a constraint to analysis: mapped thematic data as a case study. In: Harvard Papers on Geographic Information Systems. Edited by G. Dutton. (Reading, MA: Addison-Wesley).

Stevens, S. S. “On the theory of scales of measurement.” *Science* 103 (2684 1946): 677 - 680.

Stinson, Craig. *Running Microsoft Windows 95*. Redmond: Microsoft Press, 1995.

Tomlin, C.D. “Digital Cartographic Modeling Techniques in Environmental Planning.” Ph.D., Yale University, 1983.

Tomlin, C. D. *Geographic Information System and Cartographic Modeling.* New York: Prentice Hall, 1989.

Udell, Jon. “Componentware.” *BYTE* 19 (5 1994): 46 - 56.

Volta, G.S., and Egenhofer, M.J. “Interaction with GIS Attribute Data Based on Categorical Coverage.” In *Spatial Information Theory: Theoretical Basis for GIS*, ed. Frank, A.U., and Campari, I. 215-247. 716. Heidelberg-Berlin: Springer Verlag, 1993.

Volta, Gary S. 1992. Categorical coverage data in geographic information systems: a model for human interaction, Master of Science Thesis, University of Maine.