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CHAPTER No.

Pragmatic Information Content – How to Measure the Information in a Route Description

Andrew U. Frank

1 INTRODUCTION

Shannon and Weaver published 1949 a breakthrough book on how to measure the information transferred over a channel. They introduced the unit *bit* as a measurement unit for information, which stands for one binary decision. This method is commonplace today and widely used to measure amounts of *data* capacity for storage devices, etc. It does, however, not assess the pragmatic *information* content of a message.

Two messages of very different data and size may communicate the same message and have therefore the same information content; we will call this the pragmatic semantics. We also know that the same message may have very different information content for different users. A theory for a measure of pragmatic information content must account for the fact that different messages may have the same content and that the same message may have different content for different recipients.

In the prototypical situation a recipient of a message uses the information to make a decision about an action. Other situations, where information is assimilated for later usage require some slight extension of the method, but always, information is only useful pragmatically when it influences a decision.

To determine pragmatic information content, the user is modelled as an algebra. All messages which lead to the same actions have the same information content, which is the minimum to determine the action. If two users differ in the action they consider, their algebras differ and therefore the information they deduce from the information content of the same message is different. Both cases are formalized in this paper with algebraic tools.

1.1 Motivation Example

A friend tells me how drive from *Kirchberg am Wechsel* to *Gloggnitz* (Figure 1)—a drive between two small towns south of Vienna (Table 1):

Table 1

Kirchberg am Wechsel to Gloggnitz
Follow the road to Otterthal
In Otterthal turn right towards Gloggnitz
Follow the road through Schlagl and Graben
Cross under the Semmering highway
Follow the road into the town of Gloggnitz

I do not fully trust his information and check with a service on the Web, which produces the following route description (Table 2):

Table 2

Your route from Kirchberg am Wechsel to Gloggnitz:
The total distance is 13.1 km.
To drive this distance will probably take 00:21 (hh:mm).

Street name	Driving Time	Route Description	Length	Distance from start
LH134\Markt	00:00	On LH134\Markt	4,1 km	4,1 km
LH134\Otterthal	00:06	Turn right on LH134\ Otterthal	6,6 km	10,6 km
LH134\Graben	00:16	Turn right on LH134\ Graben	430 m	11,0 km
LH134\Graben	00:16	Turn right on LH134\ Graben	770 m	11,8 km
B27\ Semmeringstrasse	00:18	Turn right on B27\ Semmeringstrasse	650 m	12,5 km
Hoffeldstrasse	00:19	Turn right on Hoffeldstrasse	500m	13,0 km
Sparkassenplatz	00:20	Turn right on Sparkassenplatz	50 m	13,0 km
Sparkassenplatz	00:21	Turn left on Sparkassenplatz	128 m	13,1 km

Is this the same route as described by my friend? My curiosity is started and I check two other descriptions (Tables 3 and 4):

Table 3

Start: A-2880 Kirchberg am Wechsel Destination: A-2640 Gloggnitz							
No.	State	Node	Direction	Road	km	Total km	Time
1	A	Kirchberg am Wechsel			0.0	0.0	00:00
2	A	Ramssattel			2.7	2.7	00:15
3	A	RS	Left on	B27	6.5	6.5	00:16
4		Gloggnitz			1.5	10.7	00:18
Total distance: 10,7 (km); total driving time: 00:18 (hh:mm)							

and

Table 4

Time	Total km	Description	Turn	Road
00:00	0.0	A-2880 Kirchberg am Wechsel – Markt		
00:13	6.2		Half left	
00:16	7.8		Stay left	
00:22	11.7		Turn right on	B17
00:22	12.0	A-2640 Gloggnitz		

I realize that I have received four times information to drive between the same locations—encoded in four different forms. Is it the same information? What do we mean by “the same information”? Careful analysis shows that the first two descriptions (Tables 1 and 2) give the same route and differ from the last two (Table 3 and 4). The instructions contain the same information but present it in a different form. How do we measure *pragmatic information content* for messages of different size, which lead to the same actions?

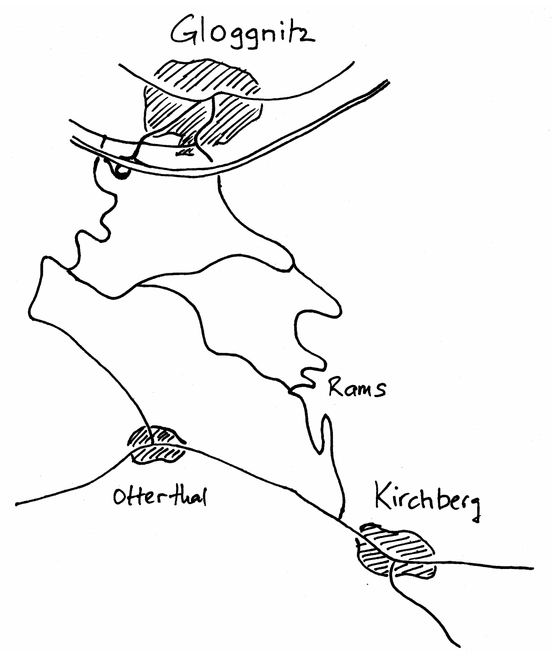

Figure 1 Map of the area

1.2 Analysis

The theory of Shannon and Weaver (1949) is widely used to measure the size of messages in storage or transfer; it measures the amount of data which is stored or transmitted in *bits*, i.e., a unit of a single binary decision. It does not measure the pragmatic information content of a message—it measures the amount of data in a message, not the effects the message has.

Table 5

Theory of pragmatic information content	
(EQ)	Two messages are equivalent when they lead to the same actions.
(SAME)	Equivalent messages of different size have the same pragmatic information content.
(DIFF)	The same message has different pragmatic information content when used in different decision contexts.

The pragmatic information content depends on the message and the situation in which the information is used to make a decision. It is therefore necessary to model the receiver of the message and the decision the message is used for making. The message itself is assumed to be a fixed artefact produced by the sender. As such its content after production does not depend on the sender anymore. Practically, the interpretation of a message by a receiver may be affected by the receiver's knowledge about the circumstances of the sender.

In this article, I suggest a formal approach to relate data to the practical situation in which it becomes information. When a message is used to decide on some action, then the message becomes information (in the sense of pragmatic semantics) and the pragmatic information content of a message can be identified and measured—*with respect to this decision context*. The information content of all equivalent messages (rule EQ) is measured as size of the minimal message necessary for the decision (rule SAME); the pragmatic information content is measured against a practical situation in which the information is used—the same message has different pragmatic information content for different users and different uses (rule DIFF). The use of the information is formalized as an algebra and the size of the minimal message is measured with the method of Shannon and Weaver (Table 5).

This article follows from ontological studies with a multi-tier ontology (Frank, 2001a; 2001c; to appear); and expands on an idea by Wittgenstein where he suggests to use games as an analogue, which abstract important properties from practical situations which are too complex to analyze. In my contribution to the Wittgenstein-Symposium 2001 (Frank, 2001b) I have explored the formalization of board games as algebras and these concepts are here extended to the formalization of a user of information in a spatial decision: driving a car on a road network is comparable to a board game. This leads me to the definition of a pragmatic information measure, which fulfils the two conditions mentioned above. A connection of this theory to game theory (von Neumann and Morgenstern, 1944) is possible.

The paper is motivated by the need to measure the information provided by Geographic Information Services, like the route planners initially shown (Krek, 2002). How should such services charge for the information they provide? By the character transmitted? By connect time?

Unfortunately, actual route descriptions as given in the examples leave many questions of a driver open. They are difficult to use and it is not clear, what their intended semantics are. In this article a formal description of semantics of standardized route descriptions are given and types of route descriptions with their semantics defined. For simplicity, I use as background instructions for navigating in a city street network already published elsewhere (Frank, 2000). To measure the pragmatic information content of other messages follows the same concepts, but results in more variability in the content, introduced by more variability in the decision the information could be used for. Different situations lead to different information extracted from the same message.

The paper is structured as follows: the next section reviews the classical theory of information measurement and the following Section 3 describes pragmatic information content measure informally. Section 4 shows how to model the decision context of a user using a message to make a decision as an algebra.

Section 5 models different user situations as algebra. Section 6 then defines a measure which satisfies the equations listed. Section 7 points to application of these ideas to the geographic information business. The concluding Section 8 summarizes the results and points to some open questions.

2 THE MATHEMATICAL THEORY OF COMMUNICATION

In their landmark contribution Shannon and Weaver have analyzed the transmission of messages over channels and how the message is affected by noise. Their measure of information is applicable to the technical level of communication. It measures the size of a message in binary decisions necessary to reconstruct the message and suggested *bit* as the fundamental unit to measure information content. This measure is widely used today and the unit bits and its multiples, i.e. Byte = 8 bits, and kilobytes, megabytes, etc. have become household words to measure the capacity of storage devices and communication channels.

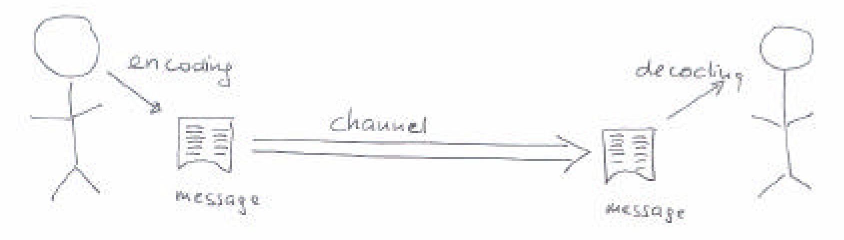

Figure 2 Transmission of a message through a channel (Shannon and Weaver, 1949)

A message of one bit is transmitted over a channel from a sender to a receiver if the sender informs the receiver about a decision between exactly two choices of equal probability; in the prototypical case the sender throws a coin and transmits the result as ‘heads’ or ‘tails’. To decide between more choices—e.g., the selection of a candidate in an election out of eight—requires three binary decisions (first to select the first or the second four, then the first or second two out of the four and then one out of the two). In general, the information content in bits is the logarithm to base 2 (logarithms dualis, ld) of the number of choices. For practical purposes the result is usually increased to the next entire number.

$$(entropy)\ H = -K \sum_i p_i * ld\ p_i \qquad (1)$$

If the choices are not of equal probability, then the information H is the weighted sum of these probabilities (entropy formula). The negative sign is necessary to convert to a positive value; notice that the probabilities p_i are all less than 1 and the ld p_i therefore negative. K is a positive constant. Shannon pointed out the relationship with similar measures in physics and suggested the term *entropy* (or uncertainty) for this property of a source of messages.

To guard against errors in transmission over noisy lines, *redundancy* is added. Redundancy can be used to reconstruct a partially transmitted text and to correct transmission errors. Typically natural language text contains considerable redundancy, estimated for English at about 50%. A text where every other character is left out can be read without much trouble.

A message can be encoded with different redundancy—usually the redundancy will be matched such that the signal and the redundancy are less than the capacity of the channel. Redundancy is measured in bits as well. The size of a message as transmitted is therefore the data content plus redundancy. Given only a message, one can measure the size of the message in bits, but not separate the data content from the redundancy. The next section discusses a method to identify pragmatic message content and separate it from redundancy.

3 PRAGMATIC INFORMATION CONTENT

Pragmatic semantics and pragmatic information content of messages must be investigated not in transmission situation as described by Shannon and Weaver (see Figure 2) but in a decision situation (Figure 3). The connection between the information in the message which is used to make a decision about some action and the decision itself needs to be considered—Shannon and Weaver's method stops when the message is correctly received.

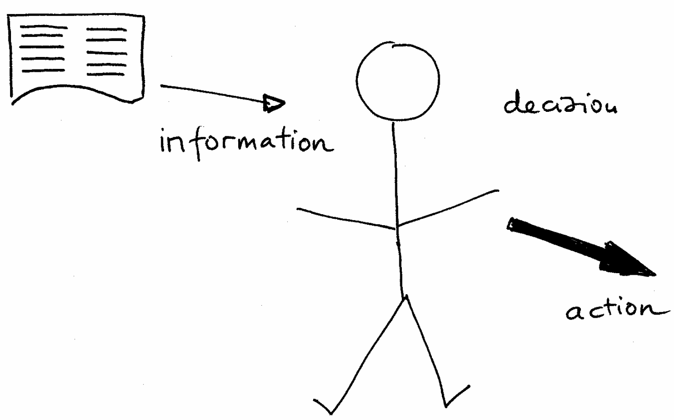

Figure 3 The decision context

Information is used to make decisions between actions—it is difficult to see another use of information. Often, we acquire information ahead of time and store, i.e. learn, facts for which we expect later a use in an expected decision situation. The determination that four messages of the initial example are essentially the same information is based on the pragmatics of 'finding my way to my friend's town'. The messages are equivalent if I find the same way to my friend's home.

A measure of pragmatic information content is different from the measure of data size of messages using the theory of Shannon and Weaver. The measure of Shannon and Weaver is not adequate for information content. For example, two of the route descriptions given initially have the same pragmatic semantics, but different message sizes. A widely held opinion therefore wants to restrict the entropy formula to technical circumstances and declares it inappropriate as a ‘real’ information measure.

3.1 Pragmatic Equivalence of Messages

Messages have the same pragmatic semantics if they lead to the same action—assuming a fixed decision situation. If I have to drive from Kirchberg to Gloggnitz, then a series of decision situations are fixed: at each intersection I have to decide which way to turn. Two of the instructions given initially, if properly interpreted, lead at these intersections to the same decisions. These instructions are therefore pragmatically equivalent.

3.2 Different Messages for Different Decision Contexts

If we give instructions, we adapt them to the person to whom we give them. Route descriptions assume that drivers have certain abilities. Some route descriptions refer to cardinal directions, most web-based ones use distances. Not all drivers are certain where the cardinal points are while driving and many ignore the odometer which would give them distance information. They cannot effectively use such instructions. Some drivers can follow a named or numbered highway through many intersections; others need instructions at each intersection. I once went in Virginia from Lee Highway 2000 to Lee Highway 10620—14 miles of winding road through many tricky intersections where I got lost more than once! Many route descriptions from the web require additional information gathered from the road signs and knowledge about the location of places mentioned on road signs and in the route description.

An instruction type is geared towards a specific decision situation, where the decision maker has determined ability and knowledge. Users with more knowledge can often use instructions prepared for less knowledgeable users, but not the reverse. For example, users with a general geographic knowledge of the area can use detailed descriptions, ignoring a large part of the message.

3.3 Pragmatic Information Content

If two messages are pragmatically equivalent—i.e., they lead to the same decision—, they have the same pragmatic information content. Even if their size, measured as size of data to be transmitted using the entropy formula, is different, the measure of pragmatic information content must be the same.

For a knowledgeable user (agent C in Figure 4) a succinct instruction is sufficient with a low pragmatic information content. If the same user is given a more detailed one, for him, the more detailed instructions have the same pragmatic semantics and therefore the same (low) pragmatic information content. For a user which requires a detailed instruction (agent B in Figure 4) the same message has a higher pragmatic information content because he has less world knowledge already available. Other agents acquire information from the environment and need therefore fewer instructions (agent A in Figure 4). For the knowledgeable user, much of the detailed message is redundant and not part of pragmatic information content—in the extreme case, where somebody knows the way from Kirchberg to Gloggnitz already, the message does not contain any new information, i.e., no pragmatic information content. The decisions taken without the instructions would be exactly the same!

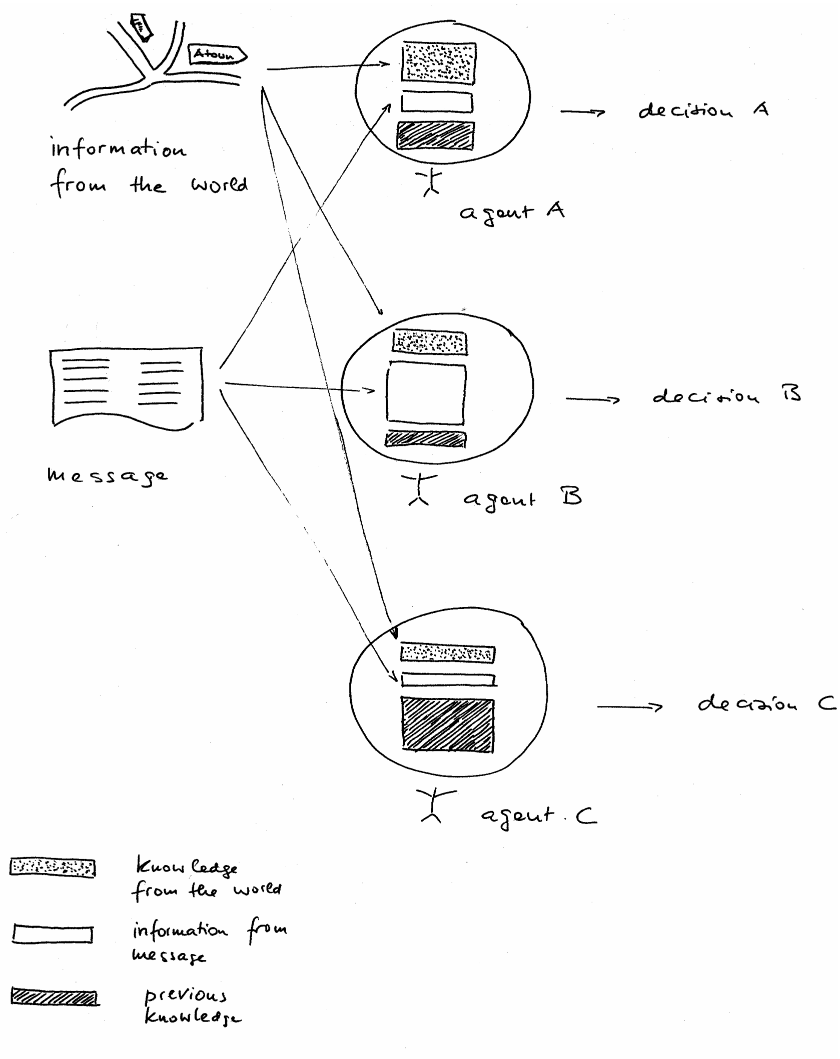

Figure 4 Three different agents use different amounts from information resources

3.4 Formal Description of Use of Information in Decision Required

Pragmatic information content can only be measured with respect to a determined decision situation and decision process. It is therefore crucial to define the decision context precisely and to assess instructions with respect to the decision context. This will be done formally in the next two sections.

3.5 Redundancy

Data in the instruction which is not required is considered redundant. The driver reaches his target without this data as well—only the necessary part is translated to information and used to make the decision. In practice, redundancy is crucial to respond to unexpected situations, missing street signs, errors in the data used to produce the route description, etc. In this article only the role played by the necessary information is investigated. The value of redundancy in the instructions needs a separate assessment.

4 A DECISION CONTEXT IS MODELED AS AN ALGEBRA

To determine information content a description of the decision situation must come first. This description explains how the instructions can be understood by a driver, i.e., the semantics of the instructions. Using agent theory, which considers autonomous agents in an environment (Ferber, 1998; Weiss, 1999), we construct a model of an agent simulating driving in a model of the street network and consider the decision this agent must make at each intersection. The agent with the operations to make the simulated moves in the street network is modelled as an algebra; the instructions must identify the operations the agent must take and provide the necessary parameters.

4.1 Agent Theory to Model the Situation

Multi-agent theory gives a framework (Ferber, 1998; Weiss, 1999) in which we can formalize decision contexts: Agents perceive an environment, make decisions based on their perception and knowledge, and carry out actions which change the environment and their position in it (Figure 5). This cycle is executed repeatedly, for example, for each instruction in a route description. For the formalization of simple decision contexts, a single agent is sufficient and other interesting concepts of multi-agent systems are not required.

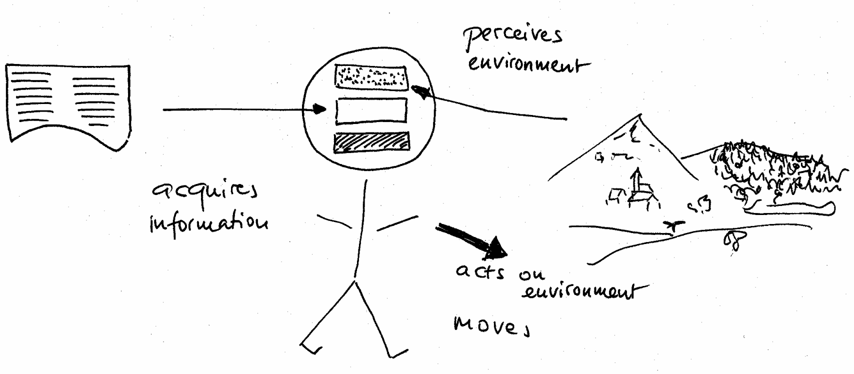

Figure 5 Cognitive agent in environment

For multi-agent programming extensive tools are available (Rao, 1996); our interest here is in building a method for measuring pragmatic information content and we revert to mathematical tools, in this case algebra. Multi-sorted algebras consist of types (mathematicians call them sorts), operations, and axioms. The operations have objects of the defined types as arguments and produce such objects; the axioms describe the outcome of the operations (Birkhoff and Lipson, 1970; Loeckx *et al.*, 1996).

Algebras have the desirable property that they are semantically self-contained and do not require other definitions; an algebra defines objects and operations completely and independently (up to an isomorphism).

4.2 Ontology: The Street Network Assumed

It is not evident, how to interpret the route descriptions shown initially and it is difficult to see if the descriptions are equivalent. One would have to follow them actually driving and then see if the path taken is the same. For the formal treatment here, I use a simplified description of a small part of downtown Santa Barbara (Figure 6); this has been used previously as a simplistic environment for map making and map interpreting agents (Frank, 2000).

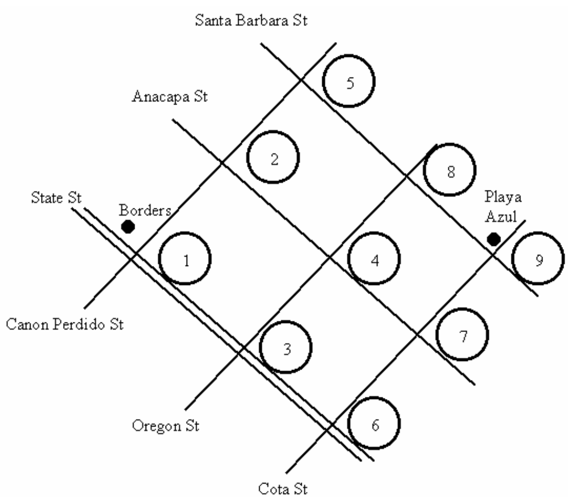

Figure 6 A small subset of streets of downtown Santa Barbara (with intersection identifiers)

The street network consists of street segments, which run from an intersection to the next. In the model the street intersections are identified with numbers. This is a simplification of the well-known TOURS model (Kuipers and Levitt, 1978; Kuipers and Levitt, 1990). The state of the world and the agent is merged in a single state variable, which maintains the complete state of the model of agent and environment.

```
class BasicDrivingAgent agent env intersection where

     startAt :: intersection -> state -> state
     isAt :: state -> intersection
     headsTo :: state -> intersection

     move :: state -> state
     turnTo :: intersection -> state -> state
```

Agents are located in this environment at a street intersection and are oriented to move to a neighbouring intersection. They can turn at an intersection to head towards a desired neighbouring intersection and they can move to the intersection they are heading towards. They are modelled after Papert’s Turtle Geometry (Abelson and diSessa, 1980; Papert and Sculley, 1980). After a move, the agent heads to the node it came from (Figure 7). This—not quite natural—behaviour leads to the smallest set of axioms for its definition; it can be defined with only four axioms (the operation *connectedIntersections* returns all nodes connected to the node given as an argument):

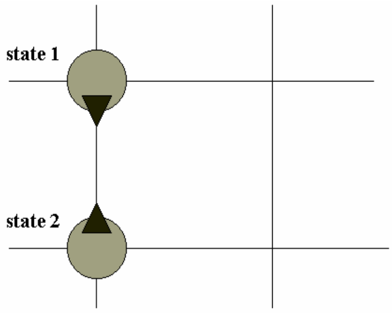

Figure 7 The position of an agent before (state1) and after a move (state2)

1. Turning does not affect the position:

   isAt (a, (turnTo (a,n,e)) = pos (a,e) (2)

2. Moving brings agent to the node that was its destination:

   isAt (a, move (a,e)) = headsTo (a,e) (3)

3. The destination after a move is the location the agent was at before the move:

   headsTo (a, move (a,e)) = isAt (a,e) (4)

4. Turning (changeDestination) makes the agent's destination the desired intersection:

   headsTo (a, turnTo (a, n, e)) =
   if n elementOf (connectedIntersections (pos (a, e) e) then n
   else error ("not a connected intersection") (5)

This model is the model of the actions a driver can take on an 'intersection by intersection' level and which are checked against the available street segments (Timpf *et al.*, 1992); drivers are restricted to advance along existing streets. The implementation of the algebra as part of an agent system (for details see Frank, 2000) together with the street network data checks the legality of all moves and calculates the result of such actions. It is a model of a physical agent moving in a street network and is not intended as a model of the human decision process. I call it therefore *basic driving agent.*

4.3 Types of Instructions

The algebra of the agent defines the instructions this agent can follow. Instructions are here understood as messages which translate 'piece by piece' into actions. Route descriptions are presented as sequences of instructions, each containing an action word, which translates to an operation, and the appropriate parameters for this action. The algebra with the axioms gives the semantics of instructions and defines which instructions are meaningful for a given agent.

For example, the basic driving agent requires the following instructions to drive from Borders (intersection 1) to Playa Azul (intersection 9) (see Figure 6):

```
startAt 1, turnTo 2, move, turnTo 4, move, turnto 7, move,
turnto 9, move
```

All instructions which are meaningful for an agent (defined as an algebra) are of the same type. Typically, all instructions prepared by one web service are of the same type; some web services offer two different instruction types—often including sketches of the intersections in the more detailed one. The four initial route descriptions are all of different types and it is therefore difficult to compare them.

4.3 Instruction Equivalence is Path Equivalence

The result of carrying out a sequence of instructions for driving between two locations is that the agent has travelled through certain street segments and has arrived at the goal location. The instructions describe the path through the network. Two sequences of instructions are equivalent if they describe the same path through the network.

A path is a sequence of locations, starting with the initial location and listing all the locations a driver passes through.

A path is a sequence of locations the agent has passed through. Two paths are equivalent if they contain the same location in the same order. Route descriptions of different types can be path equivalent; when carried out, result in the same path.

Equivalence of messages is defined as homomorphism between the algebras of the receivers; it is a well-known fact that homomorphism between algebras establishes equivalence classes (Loeckx *et al.*, 1996). All messages in the same equivalence class define the same pragmatic information.

5 DIFFERENCES IN AGENTS MODELED AS DIFFERENT ALGEBRAS

The instructions given by my friend and the instructions downloaded from the Web do not consist of instructions to move from one location to the next one, as suggested by the ‘basic driving algebra’.

For example, my friend assumes that I am able to carry out the operations:

```
followRoadTo :: location -> state -> state
turnTowards:: left/right -> location -> state -> state
followRoadThrough:: location -> state -> state
cross :: streetId -> state -> state
```

This assumes substantial commonsense reasoning, reading and interpretation of street signs; if street signs with the location names indicated are not present, I will have difficulties to follow the instructions.

For example, the ‘basic driving algebra’ of moving from location to location can only be used by a person knowing the locations which are mentioned and is clearly not realistic for route information giving. Other methods to give driving instructions rely on street names (Table 2), on location names on signs on intersections and most use turn directions.

To each instruction (type) belongs a corresponding algebra which explains how to follow these instructions. Trivially, such an algebra contains an operation 'follow one instruction line' with the data in an instruction line as arguments. In this section, different algebras, which each represent a different decision environment, are formalized.

For the following examples, instructions for a path from Borders (Intersection Canon Perdido St and State St, #1) to Playa Azul (Intersection of Santa Barbara St with Cota St, #9) are used (Figure 8). A human could give the following 'natural' route description:

1. Follow Canon Perdido Street to the East for one block,
2. Turn right and follow Anacapa Street for two blocks
3. Follow Cota Street to the East for one block

Which results in the path

```
[Intersection 1, Intersection 2, Intersection 4,
Intersection 7, Intersection 9]
```

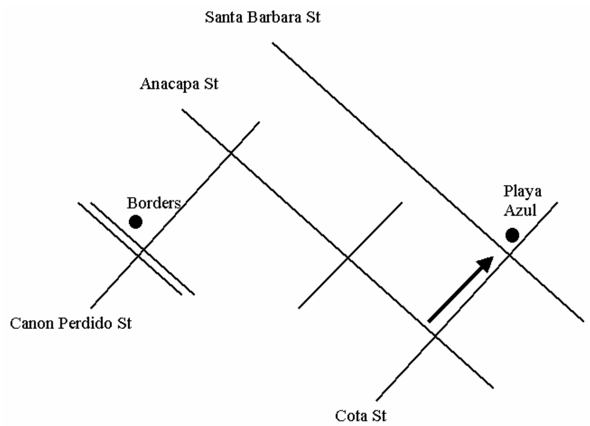

Figure 8 Sketch for path from Borders to Playa Azul

5.1 Driver "Turn and move"

In regular instructions using the Basic Driving Agent every turnTo instruction is followed by a move instruction. Merging the two to a single instruction gives ('.' is the composition operation for actions, 'a . b' means do b then a):

```
turnToAndMove : intersection -> state -> state
turnToAndMove n = move headsTo n
```

The instruction for the path in the initial example is now:

```
initialize at 1, turn to and move 2, turn to and move 4,
turn to and move 7, turn to and move 9, -> (reached 9)
```

Information content for such a description of a path, not including the information about the start node, is per segment travelled an information about the turn.

5.2 Driver “Turn left/right and move”

A driver who responds to instructions to turn left or right and then move for one segment is using the algebra:

```
turnAndMove :: LeftOrRight -> state -> state
```

The instruction for the same path is for such a driver:

```
initialize at 1 heading to 4, turn left, turn right, turn
straight, turn left -> (reached 9)
```

5.3 Driver “Turn left/right and move straight for n segments”

A driver who responds to instructions to turn left or right or to proceed for a number of segments is using an algebra like:

```
moveFor :: Integer -> state -> state
turn :: Left_Right -> state -> state
```

The instruction for the same path is for such a driver:

```
initialize at 1 heading to 4, turn left, move 1, turn
right, move 2, turn left, move 1 -> (reached 9)
```

5.4 Driver “Turn and move distance”

A driver not familiar with the environment will pay attention to the indications of the distance and use the odometer to check his movements. He can determine the cardinal directions, perhaps using a small compass. His algebra is

```
turnAngle :: Angle -> state -> state
moveDistance: Distance -> state -> state
```

5.5 Driver “Turn and move till”

This driver is familiar with the environment; in particular he recognizes some street names and is able to read other street names from the signs often found. His algebra is:

```
turn :: Left_Right -> state -> state
moveTill:   Streetname -> Turn :: Left_Right -> state ->
state
```

The interpretation by the driver “Turn and move till” requires information, which is either known to him—“information in the head” in the terminology of Donald Norman (1988)—or information he perceives from the environment—“information in the world”.

5.6 Equivalence of Instructions

A set of instructions is equivalent if they result in an equivalent path, i.e., when an agent following the instructions touches on the same locations in the same order. This can be tested with simulated execution of the instructions against a representation of the street network (e.g. on a map).

Alternatively, we can translate the different types of instructions listed above into operations of the basic driving algebra (an example was given in 5.1).

The instruction in subsection 5.1 translates to a sequence of instructions for the basic driving agent:

```
Start at 1
turnTo 2, move
turnTo 4, move
turnTo 1, move
turnTo 9, move
```

which is exactly the instruction given in subsection 4.3. The messages in subsections 5.1 and 4.3 are therefore pragmatically equivalent.

This translation was purely formal and did not require additional information. Others need information from the street network—for example, to translate left or right turns into ‘headsTo intersectionID’ operations or to translate moveDistance in simple moves along street segments from intersection to intersection.

5.7 Conclusion

The algebra which represents a decision situation defines the method how information is used pragmatically. Different decision makers with different knowledge encounter different decision situations, i.e. use different algebras for their decision. Instructions for them must be adapted to their knowledge and ability, the instructions must relate to the algebra which describes the decision context; the instructions must use the operations and their parameters according to this algebra.

6 PRAGMATIC INFORMATION CONTENT

6.1 Determination of Pragmatic Information Content

The information content in an instruction of a given type follows from the algebra:

The information content in an action

```
op :: param1 -> param2 -> state -> state
```

is estimated as

$$H = ld\ (cardinality\ domain\ param1) + ld\ (cardinality\ domain\ param2) \qquad (6)$$

To this, we have to add the information to select this operation from all the operations in the algebra (H_o = *ld (number of operations in algebra))*. There is very often only one operation and therefore H_o is 0 (*ld 1 = 0*).

This measure assumes that all combinations of input values are of equal probability (and none illegal—i.e., the function is a total function); if only for some values a valid state change is defined, then the information content is less and must be computed using the formula for entropy (in Section 2).

6.2 Property 1: Different Message, Same Information

A particular agent with a determined algebra expects instructions in the corresponding form. Most humans are versatile and can follow instructions of various types. The algebra of such a decision situation contains the 'basic driving operations' plus some additional ones, which this agent knows how to translate into the basic operations.

The size of the instructions an agent can use varies (see Section 6.1) and if the agent can respond to a number of instruction types, these form equivalence classes of instructions leading to the same actions.

The pragmatic information content for all equivalent instructions an agent in a given situation can use must be the same. Therefore, the information content is the size of smallest instructions in this equivalence class, i.e., the instructions which contain no redundancy (with respect to this agent definition). The beneficial effects of redundancy are not considered in this paper and the question is left for future work.

> The pragmatic information content is the size of the instruction without redundancy for this agent algebra.

Different messages this agent understands may have different data size, but have the same pragmatic information content, namely the data size of the smallest message. This measure is completely dependent on the abilities and knowledge of the agent (modelled as an algebra).

6.3 Property 2: Same Message, Different Information

The same message used by two different agents with different decision context may lead to very different assessment of the pragmatic information content of the message. Compare the agent above which intends to drive, with another agent, which

```
whenToLeave:: expectedArrivalTime -> lengthOfDrive
      -> departureTime
lengthOfDrive :: [dist&dirInstructions] -> lengthOfDrive
```

For this agent, a specialized message which contains only the expected driving time is pragmatically equivalent with a set of instructions, which contain the distance, which he divides by the expected average speed to calculate the driving time. The pragmatic information content is therefore *ld 120* = 7 bits, for an assumed driving time between 5 minutes and 2 hours.

7 INFORMATION BUSINESS

In this section I sketch how the theory developed here can be used to advance the information business, in particular the business with Geoinformation. In many decision situations, spatial and geographical information plays a role; it is often estimated that 80% of all decisions are influenced by or influence space. In ongoing research we develop methods to assess the value of geographic information in different decision situations as the contribution it makes to improve the decision (Krek, 2002); the assessment of information value is using the same algebraic concepts to model the decision situation as described here.

1. The description of the decision context as an algebra is first helpful for the design of the presentation of results and explanations for the user on how to interpret a route description. The ones found on the web leave considerable guesswork to the intended use. The pragmatic value of the information is therefore greatly reduced and the user will not trust information difficult to interpret.
2. The measure of pragmatic information content can be used to determine the charges for instructions provided, identifying what is information and what is redundant. For different street network parts (in town, highway, local streets between small towns) different information is necessary for navigation and what is redundant is not always the same.
3. Differential pricing is a key for an effective information business. For uses of information in decision situations which have a higher value, higher prices should be charged, but users will tend to buy information designed for other, lower value uses if they contain all necessary detail.

   For example: If one user must sketch a path for somebody in a map-like way, then instructions with cardinal directions and distances are very useful and other forms of instructions cannot be used. This user takes full advantage of the rich content and deduces higher value from the data. Another user which just uses these instructions to follow a route in familiar territory would translate the instructions in turn and move n segments, and extract only much less information. For example, most of the metric data is just redundant when one moves actually in the physical street network, which keeps drivers on the prescribed roadways. To avoid cannibalism—i.e., that high value users buy the data intended for low value applications—the route descriptions for driving should contain only very approximated cardinal directions and distances, whereas a higher value instruction for drawing sketches of path contains cardinal directions and distances with sufficient precision for the task.

8 SUMMARY

8.1 Pragmatic Information Content Is Determined with Respect to a Decision Context

> The theory of Shannon and Weaver defines a size measure for the transmission of data; pragmatic information content defines a measure for the amount of information used in a decision context.
> Two messages are pragmatically equivalent —in a determined decision context—when the decision taken is the same.

A decision situation is modelled as an algebra, where the details of the message lead to a decision.

The information content in an action

```
a :: param1 -> param2 -> state -> state
```

is estimated as

$$\begin{aligned} &H\ ld = ld\ (cardinality\ domain\ param1) \\ &+ ld\ (cardinality\ domain\ param2) + H_o \end{aligned} \tag{7}$$

where H_o is the information content to select this operation from all possible operations H_o = ld (number of operations).

The pragmatic information content for a given decision situation and user is the least amount of data necessary to make the decision. If instructions contain more data, this is redundant, for example, because it is already known by the agent or extracted by him from the real situation. If the same message is used in different decision contexts, then the above method, using a different action for one and the other context, results in different pragmatic information content

8.2 Semantics of Instructions Defined by Model of Human User

The semantics of instructions is defined by the decision context, which is a model of the human user. Agents are models of human users of information and can be modelled using algebra. The algebra defines what instructions lead to the same decisions (i.e., what instructions are equivalent). This article concentrates on the general principle of measuring the pragmatic information content and the decision contexts are used only for illustration.

8.3 Open Questions

In this article, the algebras were selected for simplicity. It is an important task, to determine what good models of human drivers are: what are the abilities of drivers to follow route descriptions. Route descriptions given in natural situations, are quite different from the route descriptions listed in this article initially. Route descriptions produced by humans contain much more landmarks:

> Drive down Reinprechtsdorferstrasse till the bright blue coloured store front of the Gazelle chain store;
> Drive along the Taborstrasse till you pass the church;
> Etc.

Messages which are larger than the minimum required for pragmatic actions contain redundancy. This is useful to guard against transmission errors, but also necessary when carrying out the instructions to cope with errors in the instructions and missing information in the world. The assessment of the value of redundancy is an important question, left for future investigations. Small differences between pragmatic information content are certainly overshadowed by the contribution produced from redundant data in unexpected situations.

Different strategies of giving and following route descriptions react differently to errors:

- some fail completely if a minimal error in the instruction is encountered;
  example: list of turns—one error and a completely different path results which does not lead to the destination.
- some rely on the receiver picking up some additional information from the world;
  example: relying on street names posted at each corner—fails if these signs are missing.
- some rely on the receiver having specific knowledge of the world.

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