Formal Specifications of Image Schemata for Interoperability in Geographic Information Systems

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Abstract

The formal specification of spatial objects and spatial relations is at the core of geographic data exchange and interoperability for GIS. It is necessary that the representation of such objects and relations comes close to how people use them in their everyday lives, i.e., that these specifications are built upon elements of human spatial cognition. Image schemata have been suggested as highly abstract and structured mental patterns to capture spatial and similar physical as well as metaphorical relations between objects in the experiential world. We assume that image-schematic details for large-scale (geographic) space are potentially different from image-schematic details for small-scale (table-top) space. This paper reviews methods for the formal description of spatial relations and integrates them in a categorical view. We give examples of image-schematic specifications for large-scale (PATH) and small-scale (CONTAINER, SURFACE) space. Such specifications should provide a foundation for further research on formalizing elements of human spatial cognition for interoperability in GIS.

1 Introduction

Exchange of data between GIS and interoperability of different vendors' GIS software are topics of enormous practical interest (Buehler and McKee 1996). Unambiguous definitions are at the core of any effort to achieve the necessary standardization that allows data exchange and cooperation of different GIS.

Standardization of technical terms and the fundamental concepts necessary to make computers interact is mostly achieved or can be achieved with current tools. The abstract behavior of computerized systems can be specified in a formal language and it requires then the checking of the compliance of the target computer system—which is by definition also a formal system—with the abstract formal system. This problem is not particular for GIS but general for all computer system standardization. The difficulties are of a practical nature and related to the lack of formal definition of most current computer languages, commercial interests in maintaining incompatible systems, and the rapid development compounded with legacy systems.

The economically important and scientifically challenging question is to describe the meaning of GIS data in terms of the real world, i.e., the so-called ”semantics problem.” What does it mean that ”P 271” is a point, ”343a” a land parcel, that building ”A1” is on parcel ”343a”, A-town is on the B-river etc., and how is this meaning communicated between systems. The naive assumption that a ”rose is a rose is a rose” (Gertrude Stein) is obviously not correct: the definitions of simple geographic properties differ from country to country, despite corresponding names (Chevallier 1981, Mark 1993, Kuhn 1994).

Image schemata describe high-level, abstract structures of common situations, most of them expressing spatial relations (Johnson 1987). Image schemata (Johnson 1987, Lakoff 1987) are the fundamental experiential elements from which spatial meaning is constructed, but so far image schemata have mostly resisted formal descriptions. This paper shows exemplar formalizations of image schemata important in the geographic context (PATH) and in table-top space (CONTAINER, SURFACE). This investigation is, therefore, part of the quest for naive or commonsense physics (Hayes 1978, Hayes 1985, Hobbs and Moore 1985) and in particular for ”Naive Geography” (Egenhofer and Mark 1995).

The next section argues why the formalization of spatial relations in geographic space is crucial for further advances in the standardization and interoperability of GIS. In Section 3 the specification of image schemata is discussed and Section 4 describes methods to formalize image schemata. Section 5 gives a comprehensive method—built upon linguistics—to discover and formally describe image schemata. Section 6 explains exemplar image schemata for geographic and table-top space (i.e., PATH, CONTAINER, SURFACE) and presents their formalizations. Section 7 presents conclusions, discusses open questions, and suggests directions for further research.

2 Formalizing Spatial Meaning

The spatial domain—in which GIS facts are situated—is fundamental for human living and one of the major sources for human experience (Barrow 1992). Human language exploits the communality of spatial experience among people and uses spatial situations metaphorically to structure purely abstract situations in order to communicate them (Lakoff and Johnson 1980, Johnson 1987). The formalization of spatial relations has, therefore, been an active area of research at least since 1989 (Mark *et al.* 1995).

Topological relations between simply connected regions were treated in (Egenhofer 1989) and extensive work has followed from this (Egenhofer 1994). Metric relations between point-like objects, especially cardinal directions (Frank 1991b, Frank 1991a, Freksa 1991, Hernández 1991) and approximate distances (Frank 1992, Hernández *et al.* 1995, Frank 1996b) were discussed. Other efforts dealt with orderings among configurations of points (Schlieder 1995) and formal descriptions of terrain and relations in terrain (Frank *et al.* 1986), but formal methods were also used to formally describe the working of administrative systems (e.g., cadastre (Frank 1996a)). Linguists have made systematic efforts to clarify the meaning of spatial prepositions (Herskovits 1986, Lakoff 1987, Herskovits 1997). However, it remains an open question how to combine these interesting results within a uniform system and to apply them systematically to other examples.

The specification of spatial relations is of great practical interest to define spatial relations in spatial query languages unambiguously; the current plethora of proposals for spatial relations to complete database query languages is useless unless the relations are formally specified (which is the case for the standard relations in SQL) (Egenhofer 1992). The formal properties are the base for query optimization. Image schemata are considered good candidates as a foundation for the formal definition of spatial relations. Kuhn has pointed out the importance of image schemata as a tool to build ”natural” (i.e., cognitively sound) user interfaces for GIS (Kuhn and Frank 1991, Kuhn 1993).

3 Specification of Image Schemata

Johnson (1987) proposes that people use recurring, imaginative patterns—so-called *image schemata*—to comprehend and structure their experiences while moving through and interacting with their environment. Image schemata are supposed to be pervasive, well-defined, and full of sufficient internal structure to constrain people’s understanding and reasoning. They are more abstract than mental pictures and less abstract than logical structures because they are constantly operating in people’s minds while people are experiencing the world (Kuhn and Frank 1991). An image schema can, therefore, be seen as a very generic, maybe universal, and abstract structure that helps people to establish a connection between different experiences that have this same recurring structure in common.

3.1 Previous Formal Description of Image Schemata

Despite efforts, success in specifying spatial image schemata has been limited. An early paper (Kuhn and Frank 1991) gave algebraic definitions for the CONTAINER (”in”) and SURFACE (”on”) schemata for a discussion of user interface design. At the level of detail and for the purpose of the paper, the two specifications were isomorphic. A recent effort by Rodríguez and Egenhofer (1997) introduced more operations and differentiated the CONTAINER schema from the SURFACE schema for small-scale space, using operations such as *remove, jerk,* and *has-_contact*, and compared the application to objects in small-scale and large-scale (geographic) space. Raubal *et al.* (1997) presented a methodology based on image schemata to structure people’s wayfinding tasks. Image schemata were represented in the form of predicates in which the predicate name referred to the image schema and the argument(s) referred to the object(s) involved in the image schema (see also Raubal 1997).

In a recent paper (Frank 1998) formal descriptions for the small-scale-space-image-schemata CONTAINER, SURFACE, and LINK were given and some of the methodological difficulties reviewed. The large-scale-space-image-schemata LOCATION, PATH, REGION, and BOUNDARY were treated in (Frank and Raubal 1998).

3.2 Definition of the Concept of an Image Schema

The concept of image schemata is not well-defined in the cognitive and linguistic literature (Lakoff and Johnson 1980, Johnson 1987, Lakoff 1987). Researchers in the past have used a working definition that implied that image schemata describe spatial (and similar physical) relations between objects. Most have concentrated on spatial prepositions like “in”, “on”, etc. and assumed that these relate directly to the image schemata (Freundschuh and Sharma 1996, Raubal *et al.* 1997, Raubal 1997).

Image schemata are seen as fundamental and independent of the type of space and spatial experience. But a single schema can appear in multiple, closely related situations. For example, “in” is used for a bowl of fruit (“Der Apfel ist in der Schale.”—“The apple is in the fruit bowl.”), but also for closed containers (“Das Geld ist im Beutel.”—“The money is in the purse.”). “Prototype effects” as described by Rosch (1973a, 1973b, 1978) also seem to apply. For example, different levels of detail can be selected to describe the same image schema.

3.3 Language Dependence of Particular Image Schemata

It is possible that image schemata provide language-independent building blocks for structure and different languages may combine the building blocks differently; the list of image schemata overlaps with Wierzbicka’s list of universal language primes (Wierzbicka 1996). The obvious differences between languages are one important point in the cultural difference that hinders the use of GIS (Campari and Frank 1995) and the problem is further aggravated by regional differences within a language.

4 Methods to Formalize Image Schemata

4.1 Predicate Calculus

Lakoff (1987) gives a definition of the CONTAINER schema using predicate calculus. In theory, predicate calculus has all the expressive power necessary, but it is practically limited by the frame problem, which makes succinct definition for changes impossible (Hayes 1977, McCarthy 1985). McCarthy (1980, 1986) proposed situation calculus with circumscription as an extension of the logical theory to overcome this limitation.

4.2 Relations Calculus

The behavior of topological relations (Egenhofer 1994, Papadias and Sellis 1994), but also cardinal directions and approximate distances (Hernández *et al.* 1995, Freksa 1991, Frank 1992, Frank 1996b) can be analyzed using the relations calculus (Schroeder 1895, Tarski 1941, Maddux 1991). Properties of relations are described as the outcome of the combination (the “;” operator) of two relations. The description abstracts away the individuals related (in comparison to the predicate calculus) and gives a simple algebra over relations. This leads to succinct and easy-to-read tables, as long as the combination of only a few relations is considered.

a (R;S) c = aRb and bSc

for example: North;NorthEast = {North or NorthEast}

meet;inside = {inside, covered, overlap}

4.3 Functions

Functions are more appropriate to capture the semantics of image schemata with respect to operations. Relation composition is replaced by function composition (the "." operator). In order to use this notation flexibly, a "curried" form of function writing must be used (Bird and Wadler 1988, Bird and Moor 1997).

$$f \, . \, g\ (x) = f\ (g\ (x)).$$

Function composition can be described by tables as well, but these grow even faster than relation composition tables. Axiomatic descriptions as algebras are more compact but also more difficult to read.

4.4 Model Based

A model of the scene is constructed and used for reasoning (there is some evidence that this is also one of the methods humans apply (Knauff *et al.* 1995)). A fundamental set of operations to construct any possible state of the model and a sufficient number of "observe" operations to differentiate any of these states are provided. In addition, more complex operations can be constructed using the given operations.

The simplest model is to use the constructors of the scene directly and to represent each scene as the sequence of constructors which created it (Rodríguez 1997). This gives a (possibly executable) model for functional or relation oriented description.

Such models can be ontological—modeling some subset of the existing world—or they can be epistemological—modeling exclusively the human conceptualization of the world. More than one epistemological view can follow from an ontological model.

4.5 Tools Used

Formal specifications written and checked only by human minds must be regarded with great skepticism: humans are not particularly apt in finding errors in formal descriptions. For effective work, formal (computerized) tools must be used. Two types have been used: Logic-based languages (e.g., Prolog (Clocksin and Mellish 1981)), used for the definition of spatial terminology (Frank *et al.* 1986) and for spatial relations calculus (Egenhofer 1989). Logic-based systems must use "extralogical" operations when change is considered (*assert* and *retract* in Prolog). Recently, functional languages (Bird and Wadler 1988) have been advocated (Frank 1994, Kuhn and Frank 1997), especially Haskell (Peterson *et al.* 1997) and Gofer (Jones 1991, Jones 1994). Allegories (a special kind of categories) provide the theoretical structure to unify the two approaches (Bird and Moor 1997).

5 A Linguistic Method to Discover and Describe Image Schemata

Mark and Frank (1996) showed how image schemata can be deduced from natural-language expressions describing geographic situations. The image schema that has been in the speaker’s mind while making a statement can be inferred from the preposition (e.g., in, on, under) used (Mark 1989). The same approach was also used by Freundschuh and Sharma (1996), Raubal *et al.* (1997), and Frank (1998). A number of restrictions and assumptions are necessary to make progress with this line of investigation:

5.1 Operational Definition of Image Schemata

As an operational definition of image schemata we consider spatial situations as image schemata if they can be used as a source domain for metaphorical transfer to some target domain; this demonstrates that a commonly understood structural content, that is independent of the specific situation, exists.

5.2 Assumption of Polysemy

A single word may have multiple meanings (e.g., the English word ”spring” can be the verb ”to jump”, a season, a source, etc.). We assume that polysemy helps to initially separate what are potentially different meanings of a word for formalization. If the meanings are the same after formal description is achieved, the assumed polysemy can be dropped.

5.3 Exclusion of Partial Spatial Relations

Spatial relations may be partial: a pen may be partially on a sheet of paper, a city partially in one, partially in another state or country (e.g., Niagara Falls is a city both in Canada and the U.S.A.). At the present time such situations are excluded from consideration and their analysis is postponed. Ongoing work by Egenhofer (Rashid *et al.* 1998) to differentiate situations with the same topology by metric measures characterizing the degree of overlap etc. may answer these questions.

5.4 Restriction to a Single Level of Detail and Abstraction

The level of abstraction differs depending on the requirements of the situation (Timpf *et al.* 1992, Voisard and Schweppe 1994, Voisard and Schweppe 1997). These multiple levels of detail play an especially important part in geographic space and make the specification of image schemata difficult. Level of detail may be spatial subdivision, may be the consideration of additional rules, or may be the subdivision of categories into subcategories (Jordan *et al.* 1998, Giunchiglia and Walsh 1992). All these effects are excluded from this investigation.

5.5 Different image-schematic details for geographic and table-top spaces

We assume that image-schematic details for geographic space are separate from image-schematic details for small-scale space (Montello 1993, Couclelis 1992). Some of Johnson's (1987) suggested image schemata use terminology from geographic space (e.g., PATH), others suggest that the same image schema (e.g., SURFACE) is used for different types of spaces. If the same terminology is used, we assume here—for methodological reasons—polysemy.

5.6 Concentration on a Single Language and Epistemology

The examples given here are in German (with English translations) as this is the authors' native language; the results can be compared with the English language situation and some differences observed (Herskovits 1986, Montello 1995). The language examples are the driving force here and the concentration is on the epistemology.

6 Formal Specifications of Image Schemata for Large-Scale and Small-Scale Space

This section shows examples of image-schematic formalizations for geographic and table-top space.

6.1 Example of Formal Image-Schematic Specification for Large-Scale Space

The subset of reality considered here consists of some geographic-space-objects plus the immediate relations between them. Our example focuses on the path schema: A PATH connects locations and consists of a starting point, an endpoint and a connection between them, as defined in (Johnson 1987).

Geographic space is rich in derived spatial relations and only when we consider the movement of persons in the landscape preconditions and changes in the scene—of which the person is part of—must be discussed. The relations among geographic objects are static and can, therefore, be formalized with predicate calculus. For each given relation, a converse relation exists. Relations are written in a prefix notation (similar to a predicate). Path (a, b) means there exists a path from a to b. This world is closed in the logical sense (Reiter 1984): everything is known about the scene and what is not known can be assumed to be false. In particular, there are no unknown objects, all objects have different names, and all relations are known or inferred from the image schemata. This is all typically not the case in natural human discourse.

6.1.1 Location and Relations between Places

A path connects places. We differentiate between the simple “direct path” and the “indirect path”, which consists of a sequence of “direct paths.” At this level, different types of paths are not differentiated (i.e., no particulars of railways, highways, etc. are considered).

A direct path connects locations directly, without any intervening location (at the level of detail considered). A direct path has a start and an end location (Figure 1a). At this level of detail there is no need to model path as an object, just as a relation between two places (path (a, b)).

Es gibt einen Weg von Wien nach Baden.
There is a path from Vienna to Baden.

For this environment (but not for a city with one-way streets) the path relation is symmetric (Figure 1b):

path (a, b) <=> path (b, a)

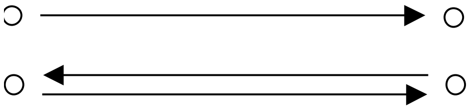

Figure 1a,b: Direct path and symmetry of path relation.

Path is its own converse relation:

Du kannst von Baden nach Wien fahren und am Abend wieder zurück.
You can drive from Baden to Vienna, and back in the evening.

conv (path (a, b)) = path (b, a) = path (a, b)

It is derived from a non-redundant base relation as the symmetric completion.

An indirect (transitive) path (ind-path) connects two locations through a sequence of direct-path-relations, such that the end location of one direct path is the start location of the next path (Figure 2).

ind-path (a,b) = [path (a, a1) & path (a1, a2) & path (a2, ...) & ... & path (..., bn) & path (bn, b)]
conv (ind-path) = ind-path

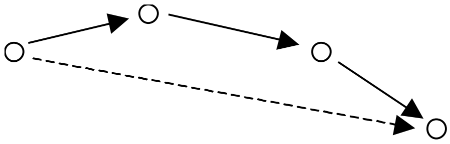

Figure 2: Indirect path.

The indirect path is derived using transitive closure. The details of the algorithm are particular to deal with cyclic and bi-directional graphs as formed by path networks and well known as shortest path algorithm (Dijkstra 1959, Sedgewick 1983).

6.1.2 Persons (and Other Autonomous and Movable Objects)

Persons move to places and are then “in” the place, unless they move further:

Er ist nach Györ gefahren, jetzt wartet er dort auf dich.
He went to Györ, now he is waiting there for you.

scene2 = move (place1, scene1) => isIn (place2, scene2)

If a person is found “in” place p1 at time t1 and place p2 at time t2 one can deduce a move (Figure 3):

Simon war letzte Woche in der Steiermark, jetzt ist er wieder in Wien.—Ist er am Samstag oder am Sonntag nach Hause gefahren?
Last week Simon was in Styria, now he is back in Vienna.—Did he drive home on Saturday or Sunday?—(move inferred in the time in-between)

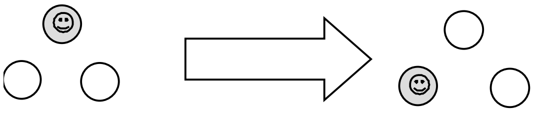

Figure 3: Move.

To move requires for a person some preconditions, unestablishes (retracts) some facts, and establishes new facts:

move (p, a, b): in (p, a) & path (a, b)

unestablish (in (p, a)), establish (in (p, b))

A person cannot move from one place to another unless there is a path:

Du kannst von Baden nicht direkt nach Schwechat fahren, du musst über Wien fahren.
You cannot drive directly from Baden to Schwechat, you have to go through Vienna.

If the person is at an unspecified location within a region, then it is only required that there is a path from every location in this region to the target.

6.1.3 Formal Executable Model

A formal, executable model for a complete set of relations has been written in a functional programming language. The difficulties of coding have mostly to do with finding consistent conventions to name all the relations. Most rules can be written as equations between relations and relation transforming functions (i.e., point-free in the categorical sense (Bird and Moor 1997)) and the formulae are valid for any scene.

6.2 Example of Formal Image-Schematic Specification for Small-Scale Space

In the second case study—table-top space—we concentrate on the affordability of movement. Again, for each relation we have a converse *(a (conv Rel) b = b Rel a).* The spatial relations and their converses are interpreted as Boolean functions *fRel (a, b) -> Bool,* or functions which return for an object the relatum *fRel (a) -> b = a Rel b.* We say that an object *participates* in a spatial relation *Rel* if the corresponding *fRel* returns an object (this is equivalent to *∃b: a Rel b*).

We consider the following image schemata for small-scale (table-top) space:

- CONTAINER: A CONTAINER has an inside, an outside, and a boundary.
- SURFACE: The SURFACE schema is used to describe the support of objects.

We focus on the common-sense spatial reasoning conclusions from the relations “in” (CONTAINER) and “auf” (SURFACE) between an object and a relatum, and the operations to establish such relations (*moveIn, moveAuf*).

6.2.1 “In” Blocks Target of Movement

An object cannot be moved to a target if this is already in another object (Figure 4). This is justified by situations as:

> *x 'in' y (in scene) => blocked (move z into x (in scene))*
>
> *Du musst den Beutel zuerst aus der Tasche nehmen, bevor du die Münze hineingeben kannst.*
> *You must take the purse out of the pocket to put the coin in.*

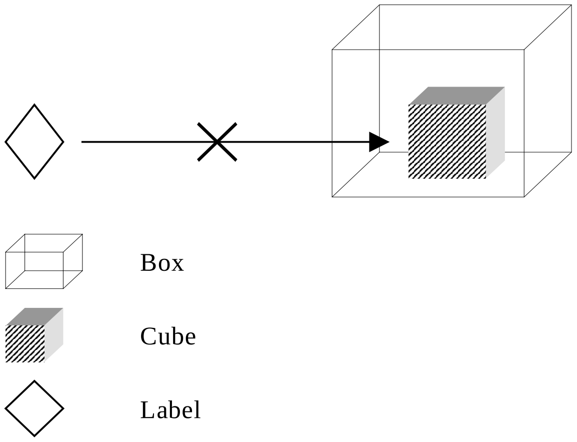

Figure 4: Cube is in box. Not permit: paste label on cube.

6.2.2 Converse of “auf” Blocks Object of Movement

“Auf” blocks the movement of the supporting object (Figure 5). It cannot be moved unless the object “auf” it is removed.

x ’auf’ y (in scene) => blocked (move y in scene)

Teller und Gläser sind auf dem Tisch. Wir müssen den Tisch zuerst abräumen,
bevor wir ihn auf die andere Seite des Zimmers bringen können.
Plates and glasses are on the table. We have to remove all objects from the table,
before we can move it to the other side of the room.

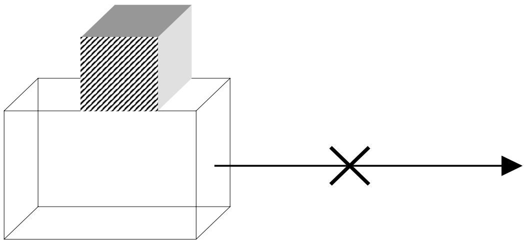

Figure 5: Cube is on (auf) box. Not possible: move box.

6.2.3 Formal Model

A function composition model can be constructed and the rules listed are directly coded. The central operation “move” for one example given with the arguments: relation type, object, target, scene is shown below;

```
move i a b s =
if fRel’ In b s      -- rule 7.2 : in blocks target of movement
   then error ("in blocked: already in")
move i a b s
```

7 Conclusions, Open Questions, and Future Work

Formal descriptions of spatial relations as encountered in everyday life are very important for GIS. They can be used to formally define query language predicates and to optimize the execution of spatial queries. They are crucial for the specification of spatial data exchange formats and GIS interoperability standards.

Most previous efforts to analyze spatial relations have used relation calculus and have concentrated on spatial relations which are amenable to this treatment. The extension of relation calculus to a function calculus is discussed here, linking two previously unconnected tools. The two tools are not as different and their conceptual merging is in category theory (Barr and Wells 1990, Herring *et al.* 1990, Asperti and Longo 1991, Walters 1991). Function composition tables can be used similarly to relation composition tables; they show patterns which can then be succinctly formulated as rules.

In this paper we applied a linguistic method based on prepositions to describe image schemata. We showed examples for large-scale and small-scale space and presented them in a formal way. With this approach the common-sense knowledge about the environment considered is captured in a strong set of implications following from individual relations.

Many open questions still remain and should be considered for further research:

7.1 Methodological

The method used here is borrowed from linguistics. For linguistic demonstrations, a single utterance which is acceptable by a native speaker is sufficient to demonstrate the existence of a construct. Is a single commonsense reasoning chain as given here sufficient? It documents that at least one situation exists where the suggested spatial inference is made—thus it demonstrates at least one aspect of a spatial relation in (one human’s) cognition. In order to verify the universality of such spatial inference mechanisms, extended human subjects testing among people with different native languages is needed.

7.2 Language-Independent Primitives

Can language-independent primitives be identified (in the sense of Wierzbicka (1996))? Investigation of the same domain by researchers with different mother tongues would be necessary (or at least a collection of the related natural language descriptions). For the domains and examples here, the spatial inferences are also correct in the translations, but the use of spatial prepositions differ between German and English.

7.3 Relation Between Relations and Functions

The use of category theory to establish a common theoretical ground for a relation (static) view and a function (dynamic) view is new and must be further explored. A category can be constructed over both functions and relations (Bird and Moor 1997). It is also possible to map relations into functions (aRb --> f (a,b) : Bool) and functions into a relation (f (a):: b --> aRb) as was used here. Certain formalizations seem to be easier in the one, others in the other.

In any case, the formulae must be interpreted with respect to an "environment" of the facts (we used the term "scene"). Functions like "move" change the scene. We currently experiment with monads—a device from category theory—to have the environment implicit in the formulae and, therefore, reduce the complexity of formalization (Wadler 1997, Liang *et al.* 1995).

7.4 Composition and Interaction of Image Schemata

The combination of multiple image schemata and the interaction of image schemata with object's properties must be further explored. For an object to move along a path, it must be of the appropriate kind (only trains run along railway lines, cars cannot follow a foot path, etc., and similar restrictions apply in other cases). Possibly, the current approach trying to capture image schemata with the definition of spatial prepositions is too limited. Raubal *et al.* (1997) used prepositions and semantic connotation to investigate superimpositions of image schemata. Another interesting approach is to look at affordances. Affordances seem to be closely related to image schemata because both of these concepts help people to understand a spatial situation in order to know what to do (Gibson 1979). Affordances might be operational building blocks of image schemata but further research in this area is needed (Jordan *et al.* 1998).

7.5 Are Image Schemata the Smallest Constituent Parts of Spatial Cognition?

Are image schemata the atoms of spatial cognition or are there smaller semantic units from which image schemata can be composed? It appears as if these were smaller pieces from which the more complex image schemata could be built, but one could also argue that these are the image schemata proper.

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References

A. Asperti and G. Longo (1991) *Categories, Types and Structures - An Introduction to Category Theory for the Working Computer Scientist*. The MIT Press, Cambridge, Mass.

M. Barr and C. Wells (1990) *Category Theory for Computing Science*. Prentice Hall.

J. Barrow (1992) *Pi in the Sky. Counting, Thinking and Being.*

R. Bird and O. d. Moor (1997) *Algebra of Programming*. Prentice Hall, London.

R. Bird and P. Wadler (1988) *Introduction to Functional Programming*. Prentice Hall International, Hemel Hempstead (UK).

K. Buehler and L. McKee, Eds. (1996) *The OpenGIS Guide - An Introduction to Interoperable Geoprocessing*. The OGIS Project Technical Committee of the Open GIS Consortium, Wayland, MASS.

I. Campari and A. Frank (1995) Cultural Differences and Cultural Aspects in GIS. in: T. Nyerges, D. Mark, R. Laurini, and M. Egenhofer (Eds.), 83, pp. 249-266, Kluwer Academic Publishers, Dordrecht, The Netherlands.

J. Chevallier (1981) Land Information Systems - a global and system theoretique approach. in: *FIG International Federation of Surveyors*, Montreux, Switzerland, pp. paper 301.2.

W. Clocksin and C. Mellish (1981) *Programming in Prolog*. LB-Bibli//731829, Springer-Verlag //279

H. Couclelis (1992) People Manipulate Objects (but Cultivate Fields): Beyond the Raster-Vector Debate in GIS. in: A. Frank, I. Campari, and U. Formentini (Eds.), Theories and Methods of Spatio-Temporal Reasoning in Geographic Space. Lecture Notes in Computer Science 639, pp. 65-77, Springer-Verlag, Berlin.

E. Dijkstra (1959) A note on two problems in connection with graphs. Numerische Mathematik (1): 269-271.

M. Egenhofer (1989) *Spatial Query Languages*. Ph.D., University of Maine.

M. Egenhofer (1992) Why not SQL! *International Journal of Geographical Information Systems* 6(2): 71-85.

M. Egenhofer (1994) Deriving the Composition of Binary Topological Relations. *Journal of Visual Languages and Computing* 5(2): 133-149.

M. Egenhofer and D. Mark (1995) Naive Geography. in: A. Frank and W. Kuhn (Eds.), *Spatial Information Theory - A Theoretical Basis for GIS. Lecture Notes in Computer Science* 988, pp. 1-15, Springer-Verlag, Berlin.

A. Frank (1991a) Qualitative Spatial Reasoning with Cardinal Directions. in: *7. Österreichische Artificial-Intelligence-Tagung*, Wien, Österreich.157-167.

A. Frank (1991b) Qualitative Spatial Reasoning about Cardinal Directions. in: D. Mark and D. White (Eds.), *Auto-Carto 10, ACSM-ASPRS*, Baltimore, pp. 148-167.

A. Frank (1992) Qualitative Spatial Reasoning about Distances and Directions in Geographic Space. *Journal of Visual Languages and Computing* 1992(3): 343-371.

A. Frank (1994) *Qualitative temporal reasoning in GIS - Ordered time scales*. Technical University Vienna, Dept. of Geoinformation, Technical Report.

A. Frank (1996a) An object-oriented, formal approach to the design of cadastral systems. in: M. Kraak and M. Molenaar (Eds.), *7th Int. Symposium on Spatial Data Handling, SDH '96*, Delft, The Netherlands, pp. 5A.19-5A.35.

A. Frank (1996b) Qualitative spatial reasoning: cardinal directions as an example. *IJGIS* 10(3): 269-290.

A. Frank (1998) Specifications for Interoperability: Formalizing Spatial Relations 'In', 'Auf' and 'An' and the Corresponding Image Schemata 'Container', 'Surface' and 'Link'. in: *1. Agile-Conference,* ITC, Enschede, The Netherlands.

A. Frank and M. Raubal (1998) Specifications for Interoperability: Formalizing Image Schemata for Geographic Space. in: *8th Int. Symposium on Spatial Data Handling, SDH'98*, Vancouver, Canada.

A. Frank, B. Palmer, and V. Robinson (1986) Formal methods for accurate definition of some fundamental terms in physical geography. in: D. Marble (Ed.), *Second International Symposium on Spatial Data Handling*, Seattle, Wash., pp. 583-599.

C. Freksa (1991) Qualitative Spatial Reasoning. in: D. Mark and A. Frank (Eds.), *Cognitive and Linguistic Aspects of Geographic Space*. *NATO ASI Series D: Behavioural and Social Sciences* pp. 361-372, Kluwer Academic Press, Dordrecht, The Netherlands.

S. Freundschuh and M. Sharma (1996) Spatial Image Schemata, Locative Terms and Geographic Spaces in Children's Narratives: Fostering Spatial Skills in Children. *Cartographica, Monograph 46, Orienting Ourselves in Space* 32(2): 38-49.

F. Giunchiglia and T. Walsh (1992) *A Theory of Abstraction*. Istituto per la Ricerca Scientifica e Tecnologica, Trento, Italy, Technical Report Technical Report 9001-14.

P. Hayes (1977) In Defense of Logic. *IJCAI* 1: 559 - 565.

P. Hayes (1978) The Naive Physics Manifesto. in: D. Mitchie (Ed.), *Expert Systems in the Microelectronic Age*. pp. 242-270, Edinburgh University Press, Edinburgh.

P. Hayes (1985) The Second Naive Physics Manifesto. in: J. Hobbs and R. Moore (Eds.), *Formal Theories of the Commonsense World*. pp. 1-36, Ablex Publishing Corp., Norwood, N.J.

D. Hernández (1991) Relative Representation of Spatial Knowledge: The 2-D Case. in: D. Mark and A. Frank (Eds.), *Cognitive and Linguistic Aspects of Geographic Space: An Introduction*. pp. 373-386, Kluwer Academic, Dordrecht.

D. Hernández, E. Clementini, and P. Di Felice (1995) Qualitative Distances. in: A. Frank and W. Kuhn (Eds.), *Spatial Information Theory - A Theoretical Basis for GIS (Proceedings of the International Conference COSIT'95). Lecture Notes in Computer Science* 988, pp. 45-57, Springer-Verlag, Berlin-Heidelberg.

J. Herring, M. Egenhofer, and A. Frank (1990) Using Category Theory to Model GIS Applications. in: K. Brassel (Ed.), *4th International Symposium on Spatial Data Handling*, Zurich, Switzerland, pp. 820-829.

A. Herskovits (1986) *Language and Spatial Cognition - An Interdisciplinary Study of the Propositions in English*. Cambridge University Press, Cambridge UK.

A. Herskovits (1997) Language, Spatial Cognition, and Vision. in: O. Stock (Ed.), *Spatial and Temporal Reasoning*. Dordrecht, Kluwer Academic Publishers, pp: 155-202.

J. Hobbs and R. Moore, Eds. (1985) *Formal Theories of the Commonsense World*. Ablex Series in Artificial Intelligence Ablex Publishing Corp., Norwood, NJ.

M. Johnson (1987) *The Body in the Mind: The Bodily Basis of Meaning, Imagination, and Reason*. University of Chicago Press, Chicago.

M. Jones (1991) *An Introduction to Gofer*. Yale University, Technical Report.

M. Jones (1994) *The Implementation of the Gofer Functional Programming System*. Yale University, Technical Report YALEU/DCS/RR-1030.

T. Jordan, M. Raubal, B. Gartrell, and M. Egenhofer (1998) An Affordance-Based Model of Place in GIS. in: *8th Int. Symposium on Spatial Data Handling, SDH'98*, Vancouver, Canada.

M. Knauff, R. Rauh, and C. Schlieder (1995) Preferred mental models in qualitative spatial reasoning: A cognitive assessment of Allen's calculus. in: *17th Annual Conference of the Cognitive Science Society*, Hillsdale, NJ, pp. 200-205.

W. Kuhn (1993) Metaphors Create Theories for Users. in: A. Frank and I. Campari (Eds.), *Spatial Information Theory*. *Lecture Notes in Computer Science* 716, pp. 366-376, Springer.

W. Kuhn (1994) Defining Semantics for Spatial Data Transfers. in: T. Waugh and R. Healey (Eds.), *6th International Symposium on Spatial Data Handling*, Edinburgh, UK, pp. 973-987.

W. Kuhn and A. Frank (1991) A Formalization of Metaphors and Image-Schemas in User Interfaces. in: D. Mark and A. Frank (Eds.), *Cognitive and Linguistic Aspects of Geographic Space*. *NATO ASI Series* pp. 419-434, Kluwer Academic Publishers, Dordrecht, The Netherlands.

W. Kuhn and A. Frank (1997) The Use of Functional Programming in the Specification and Testing Process. in: M. Goodchild, M. Egenhofer, R. Fegeas, and C. Kottman (Eds.), *Int. Conference and Workshop on Interoperating Geographic Information Systems*, Santa Barbara, CA, USA (3.-6. Dec. 1997).

G. Lakoff (1987) *Women, Fire, and Dangerous Things - What Categories Reveal about the Mind*. The University of Chicago Press, Chicago.

G. Lakoff and M. Johnson (1980) *Metaphors We Live By*. University of Chicago Press, Chicago.

S. Liang, A. Hudak, and A. Jones (1995) Monad transformers and modular interpreters. in: *ACM Symposium on Principles of Programming Languages*.

R. Maddux (1991) The origin of relation algebras in the development and axiomatization of the calculus of relations. *Studia Logica* 50(3-4): 421-455.

D. Mark (1989) Cognitive Image-Schemata for Geographic Information: Relations to User Views and GIS Interfaces. in: *GIS/LIS'89*, Orlando, Florida, November 1989, pp. 551-560.

D. Mark (1993) Toward a Theoretical Framework for Geographic Entity Types. in: A. Frank and I. Campari (Eds.), *Spatial Information Theory: Theoretical Basis for GIS*. *Lecture Notes in Computer Science* 716, pp. 270-283, Springer Verlag, Heidelberg-Berlin.

D. Mark, D. Comas, M. Egenhofer, S. Freundschuh, M. Gould, and J. Nunes (1995) Evaluating and Refining Computational Models of Spatial Relations Through Cross-Linguistic Human-Subjects Testing. in: A. Frank and W. Kuhn (Eds.), *Spatial Information Theory-A Theoretical Basis for GIS*. *Lecture Notes in Computer Science* 988, pp. 553-568, Springer, Berlin-Heidelberg-New York.

D. Mark and A. Frank (1996) Experiential and Formal Models of Geographic Space. *Environment and Planning, Series B* 23: 3-24.

J. McCarthy (1980) Circumscription - A Form of Non-Monotonic Reasoning. *Artificial Intelligence* 13: 27-39.

J. McCarthy (1985) Epistemological problems of artificial intelligence. in: R. Brachman and H. Levesque (Eds.), *Readings in Knowledge Representation*. pp. 24 - 30, Morgan Kaufman Publishers, Los Altos, CA.

J. McCarthy (1986) Applications of Circumscription to Formalizing Common Sense Knowledge. *Artificial Intelligence* 28: 89-116.

D. Montello (1993) Scale and Multiple Psychologies of Space. in: A. Frank and I. Campari (Eds.), *Spatial Information Theory: Theoretical Basis for GIS. Lecture Notes in Computer Science* 716, pp. 312-321, Springer Verlag, Heidelberg-Berlin.

D. Montello (1995) How Significant are Cultural Differences in Spatial Cognition? in: A. Frank and W. Kuhn (Eds.), Spatial Information Theory-A Theoretical Basis for GIS. Lecture Notes in Computer Science 988, pp. 485-500, Springer, Berlin-Heidelberg-New York.

D. Papadias and T. Sellis (1994) A Pictorial Language for the Retrieval of Spatial Relations from Image Databases. in: 6th International Symposium on Spatial Data Handling (SDH), Edinburgh, UK.

J. Peterson, K. Hammond, L. Augustsson, B. Boutel, W. Burton, J. Fasel, A. Gordon, J. Hughes, P. Hudak, T. Johnsson, M. Jones, E. Meijer, S. Jones, A. Reid, and P. Wadler (1997) The Haskell 1.4 Report. *http://haskell.org/report/index.html*

A. Rashid, B. Shariff, M. Egenhofer, and D. Mark (1998) Natural-language spatial relations between linear and areal objects: the topology and metric of English-language terms. *IJGIS* 12(3): 215-246.

M. Raubal (1997) *Structuring Space with Image Schemata*. Master's Thesis, University of Maine, Orono, ME, U.S.A.

M. Raubal, M. Egenhofer, D. Pfoser, and N. Tryfona (1997) Structuring Space with Image Schemata: Wayfinding in Airports as a Case Study. in: S. Hirtle and A. Frank (Eds.), *Spatial Information Theory - A Theoretical Basis for GIS (International Conference COSIT'97). Lecture Notes in Computer Science Vol.1329* 1329, pp. 85-102, Springer-Verlag, Berlin-Heidelberg.

R. Reiter (1984) Towards a logical reconstruction of relational database theory. in: M. Brodie, M. Mylopolous, and L. Schmidt (Eds.), *On Conceptual Modelling, Perspectives from Artificial Intelligence, Databases, and Programming Languages*. pp. 191-233, Springer Verlag,, New York.

A. Rodríguez (1997) *Image-Schemata-Based Spatial Inferences: The Container-Surface Algebra for Solid Objects*. Master's Thesis, University of Maine, Orono, ME, U.S.A.

M. Rodríguez and M. Egenhofer (1997) Image-Schemata-Based Spatial Inferences: The Container-Surface Algebra. in: S. Hirtle and A. Frank (Eds.), *Spatial Information Theory - A Theoretical Basis for GIS (International Conference COSIT'97). Lecture Notes in Computer Science Vol.1329* 1329, pp. 35-52, Springer-Verlag, Berlin-Heidelberg.

E. Rosch (1973a) Natural categories. *Cognitive Psychology* 4: 328 - 350.

E. Rosch (1973b) On the internal structure of perceptual and semantic categories. in: T. Moore (Ed.), *Cognitive Development and the Acquisition of Language*. pp. Academic Press, New York.

E. Rosch (1978) Principles of Categorization. in: E. Rosch and B. Lloyd (Eds.), *Cognition and Categorization*. Erlbaum, Hillsdale, NJ.

C. Schlieder (1995) Reasoning about Ordering. in: A. Frank and W. Kuhn (Eds.), *Spatial Information Theory-A Theoretical Basis for GIS*. *Lecture Notes in Computer Science* 988, pp. 341-350, Springer, Berlin-Heidelberg-New York.

E. Schroeder (1895) *Vorlesungen ueber die Algebra der Logik (Exakte Logik)*. Teubner, Leipzig.

R. Sedgewick (1983) *Algorithms*. Addison-Wesley, Reading, MA.

A. Tarski (1941) On the calculus of relations. *The Journal of Symbolic Logic* 6(3): 73-89.

S. Timpf, G. Volta, D. Pollock, and M. Egenhofer (1992) A Conceptual Model of Wayfinding Using Multiple Levels of Abstractions. in: A. Frank, I. Campari, and U. Formentini (Eds.), *Theories and Methods of Spatio-Temporal Reasoning in Geographic Space*. *Lecture Notes in Computer Science* 639, pp. 348-367, Springer Verlag, Heidelberg-Berlin.

A. Voisard and H. Schweppe (1994) A Multilayer Approach to the Open GIS Design Problem. in: N. Pissinou and K. Makki (Eds.), *2nd ACM-GIS Workshop*, New York.

A. Voisard and H. Schweppe (1997) Abstraction and Decomposition in Open GIS. *IJGIS* (special issue).

P. Wadler (1997) How to Declare an Imperative. *ACM Computing Surveys* 29(3): 240-263.

R. Walters (1991) *Categories and Computer Science*. Cambridge University Press, Cambridge, UK.

A. Wierzbicka (1996) *Semantics - Primes and Universals*. Oxford University Press, Oxford.