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Spatial rules that generate urban patterns: Emergence of the power law in the distribution of axial line length

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ABSTRACT

This paper studies emergence/generation of power law in rank-order distribution of axial line length, which is a global pattern observed in real cities, due to interaction of a set of seven simple spatial rules at a local scale. These rules and their interactions form a model expected to simulate the morphological structure of free spaces in unplanned organic pedestrian small cities. Effects of each of the seven rules are discussed through repeated simulations of eight possible combinations of the rules, using a bottom-up process. The results show that the rules generate environments with statistically stable rank-order distribution of axial line length that follows the power law. It means that the axial maps of the simulated environments have a scale-free hierarchical structure such that their distributions lean toward short axial lines. It also represents dominance of local spatial structure, as the model renders a faster rate of growth at a local scale while allowing a steady growth at a global scale.

1. Introduction

Archetype global[1] patterns exist in cities (Alexander, 2002), though each city has its own unique story (Lynch, 1981). Human interaction with the physical reality to obtain the maximum utility of living together generates these patterns. Complexity of cities makes it difficult to investigate the urban patterns and their causes objectively and usually leads us to subjective explanations, dominated by social, political, and economical issues. Objective explanation of urban patterns requires their regeneration incorporating spatial rules (Rezayan et al., 2008).

Carvalho and Penn (2004) introduce the concept of an archetype global urban pattern which shows that truncated rank-order distribution of the axial line length follows the power law. It means that the pattern is scale-free and has hierarchical structure emerged from a bottom-up process of gradual generation. Carvalho and Penn (2004) show that an increase in the exponent of the power law represents an increase in the dominance of the local spatial structure of the environment.

This paper provides an explanation for the pattern introduced by Carvalho and Penn (2004) studying emergence[2]/generation of the pattern using the city generation model proposed by Rezayan et al. (2008) as the evaluation model. The model is expected to simulate the morphological structure[3] of free spaces in unplanned[4] small cities that do not necessarily support the movement of vehicles. These are organic pedestrian cities.

As the model generates small cities, the findings of Carvalho and Penn (2004) are adopted for small cities. The paper considers un-truncated rank-order distribution of axial line length in small cities. The tendency between the exponent of the power law and the dominance of local spatial structure is shown to be reversed in small cities: a decrease in the exponent of the power law represents an increase in dominance of local spatial structure.

Section 2 describes the pattern of rank-order distribution of axial line length introduced by Carvalho and Penn (2004). Specifications of the pattern for small cities are defined in Section

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[1] "Global" refers to the large scale of space in this paper. It is used against "local" that represents the human scale of space.

[2] "Emergence" here does not represent its classical definition meaning that interactions of smaller parts of a phenomenon create synergy. It simply describes the gradual generation of complex global patterns due to simple interactions in local scale (Epstein, 2007, p. 3).

[3] The model is an abstract spatial structure of a city that represents the urban patterns.

[4] It means that there is no prior planning, i.e. no preplanning, for generation of a city and each individual plans for locating his/her building at a local scale.

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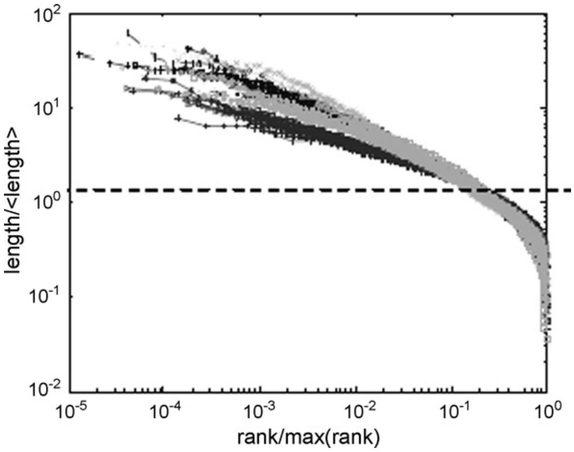

Fig. 1. The rank-order distributions of axial line length in 36 cities from 14 countries (Carvalho and Penn, 2004, p. 10). The axes have a logarithmic scale. The dashed line shows the truncation line at ln(length/<length>) = 0.2 or length/<length> = 1.221 where <length> represents average length.

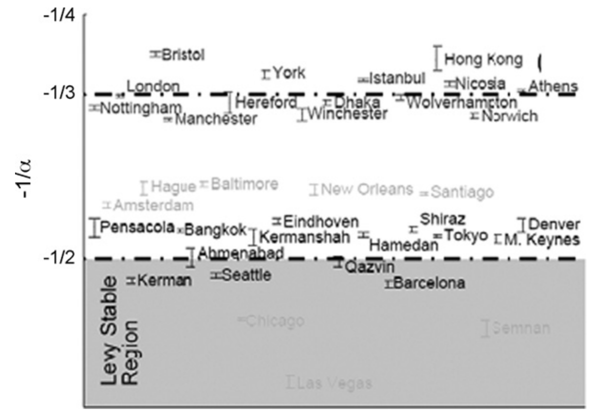

Fig. 2. Distribution of the inverse of the alpha exponent (Eq. (2)) (or zeta exponent in Eq. (1)) derived from the graph shown in Fig. 1 (Carvalho and Penn, 2004, p. 10). The stable region of the alpha exponent is shown in gray where the alpha exponent is lower than 2.

3. The remainder of the paper (Sections 4 and 5) describes the city generation model and the evaluation of the pattern of rank-order distribution of axial line length that the model generates. The evaluation is a bottom-up process based on repeated simulations of the model. For each simulation, its axial map is extracted automatically, and how its rank-order distribution of axial line length follows the power law is discussed. Both the pattern defined by Carvalho and Penn (2004) (truncated distribution) and the pattern redefined for small cities (un-truncated distribution) are investigated in the simulations. The conclusions summarize the statistical stability of the pattern generated by the city generation model and enumerate some possible applications of the model.

2. Distribution of axial line length according to power law

Hillier and Hanson (1984) define an axial line as the longest straight line that can be drawn from an arbitrary point in space. Axial lines represent convex spaces. They connect together to form an axial map that is a geometrical representation of space.

In studying the pattern of free spaces and intermittency in cities, Carvalho and Penn (2004) evaluate the rank-order distribution of axial line length (Eq. (1)) in 36 cities from 14 countries according to the power law (Fig. 1). Following the power low means that the distribution is scale-free and has hierarchical structure emerged through a bottom-up process of gradual generation.

$$\ln(size) = b - \zeta \times \ln(rank) \tag{1}$$

where *size* is the length of an axial line divided by the average lengths of axial lines (length/avg(lengths)), *rank* is the ranking of the axial line, within a list of the axial lines sorted in descending order of length, which is then divided by the maximum rank of all axial lines (rank/max(ranks)), ζ (zeta) is the slope of the trend line, and *b* is the y interception of the trend line.

Carvalho and Penn (2004) fit the Eq. (1) to the upper tail of the distribution truncated at ln(length/avg(length)) = 0.2 or length/avg(length) = 1.221 (Fig. 1) where "... data are visually the most linear (Carvalho and Penn, 2004, p. 4)". They state that "the length of urban free space structures represented by axial lines, display universal features, largely independent of city size, and is self-similar across morphologically relevant ranges of scales with [alpha] exponents 2 and 3 (Fig. 2)." (Carvalho and Penn, 2004, p. 4).

The alpha exponent is defined as the inverse of the zeta exponent (Eq. (2)).

$$\alpha = \frac{1}{\zeta} \tag{2}$$

where ζ (zeta) is the slope of the trend line shown in Eq. (1).

The alpha exponent decreases (Fig. 2) as the distribution of the axial line length leans more toward longer axial lines (Fig. 3). Carvalho and Penn (2004) discuss this effect as an increase in the dominance of the global spatial structure (Fig. 3). The increase of the alpha exponent represents an increase in the dominance of the local spatial structure. This effect can be represented using the median length of axial lines. The existence of more short axial lines moves the median length of axial lines toward their minimum length and vice versa.

Dominance of the global spatial structure increases as a city ages, and more global plans are defined and executed in the city. It is also the typical property of planned cities that are equipped by long and fast transportation routes. These two categories include many of the cities we live in now.

On the contrary, in cities which are in their early stages, and especially those which are unplanned, the dominance of local spatial structures is clearer. These kinds of cities are usually described as organic cities. They are also called pedestrian cities (Salingaros, 2004) or car-free cities (Crawford, 2002) as their structures foster pedestrian movement. This structure can also be found in surviving historical regions of cities, commercial areas, some suburbs, and urban villages (Fleming, 2000; Spears, 1997), where pedestrian passage is the main medium of movement.

The relation between dominance of the global spatial structure and the size/population of a city is not well defined. Nevertheless, considering that the dominance of the global spatial structure reduces the autonomy of the people live in cities, Christopher Alexander et al. (1977, p. 70) suggest that a city with 7000 (between 5000 and 10,000) populations can preserve the autonomy of the individuals. Besides, Crawford (2002) proposes an expandable topological structure of connected districts where each district is about 45 hectares (20% built area and 80% green area) and has around 12,000 populations, to form a car-free city that fosters the autonomy of the individuals.

3. Distribution of axial line length in small cities

Carvalho and Penn (2004) evaluate the truncated distribution of rank-order distribution of axial line length (Fig. 1). Small cities, like a village or an urban village, however, have fewer axial lines by

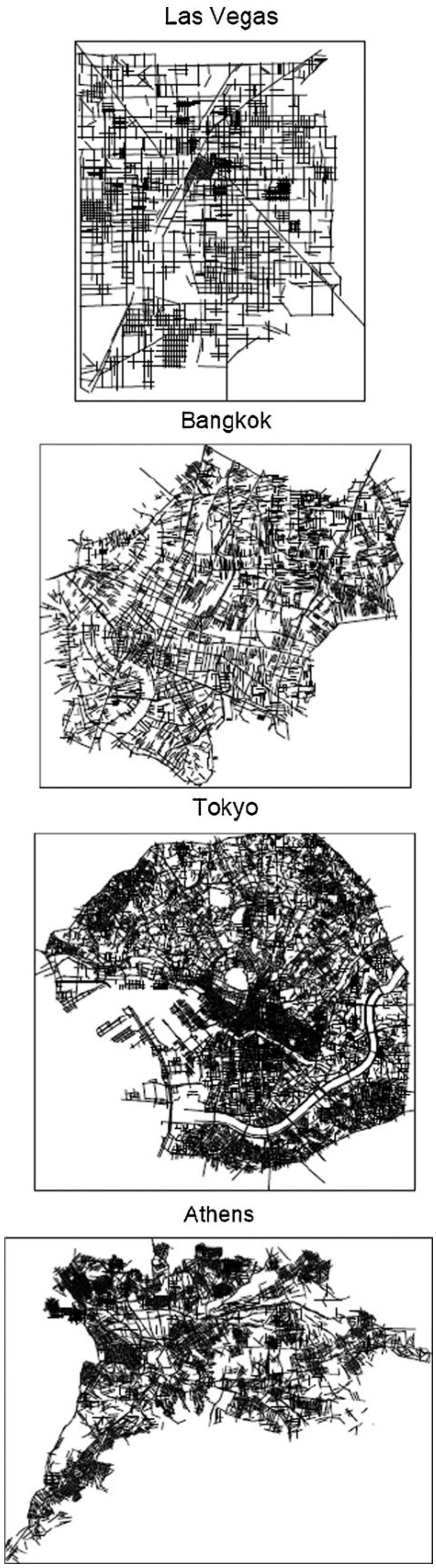

Fig. 3. Axial maps of 4 of the 36 cities studied by Carvalho and Penn (2004, p. 11). Dominance of the global rules is visible in Las Vegas, which is a planned city. Dominance of the local rules increases from top (Las Vegas) to bottom (Athens). Tokyo and especially Athens represent cities which are formed organically, and their structures are retained during the growing and planning processes carried out thus far.

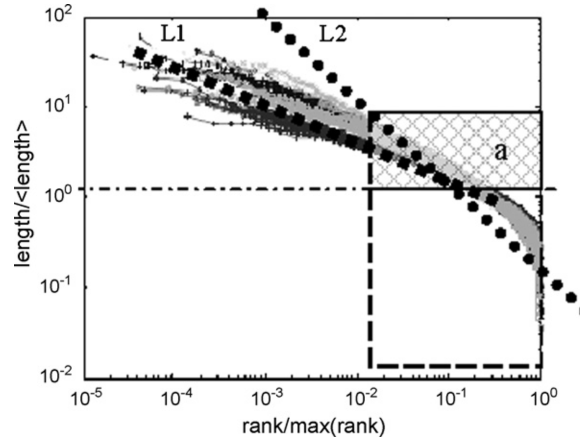

Fig. 4. The rank-order distributions of axial line length in small cities. The axes have a logarithmic scale. The window area where the rank-order distributions of axial line length for small cities reside is shown in the dashed rectangle. It includes about 100 axial lines with different lengths. The background is the rank-order distribution of axial line length in 36 cities in 14 countries (Carvalho and Penn, 2004, p. 10). The dash-dotted line shows the truncation line at ln(length/<length>) = 0.2 or length/<length> = 1.221 where <length> represents average length. The area above the truncation line which is hashed and labeled as (a) defines the truncated distribution for small cities. The bold dotted line labeled as (LI) represents a line fitted to a truncated distribution, and the bold line labeled as (L2) represents a line fitted to a non-truncated distribution.

about two orders of magnitude than the cities studied by Carvalho and Penn (2004)[5] (Fig. 4).

As shown in Fig. 4, a small amount of data remains in the truncated distribution for small cities (in the hashed rectangle labeled as (a)). Therefore the truncated distribution for small cities might not be described by the specific ranges (where the alpha exponent equals 2 and 3) and the stability region defined by Carvalho and Penn (2004) in Fig. 2. Nevertheless, the pattern of the rank-order distribution of axial line length (Fig. 4) and the effect of increase in the alpha exponent as more short axial lines emerge in the truncated distribution, still pertains.

The alpha exponent decreases toward 1, lowering the truncation line and admitting more short axial lines into the distribution (Eeckhout, 2004). It reverses the effect of the alpha exponent as a lower alpha exponent demonstrates the existence of more short axial lines and represents the dominance of a local spatial structure (Fig. 4). The bold line in Fig. 4 labeled as (L2) represents a line fitted to a non-truncated distribution. As the number of short axial lines increase, the lower tail of the distribution becomes steeper. It increases the slope of the fitted line, which is the alpha exponent of the fitted power law.

Admittance of short axial lines breaks the proportionate pattern of rank-order distribution of axial line length defined by Carvalho and Penn (2004). It generates a hierarchical structure of axial lines that its growth rate increases moving toward lower levels/shorter axial lines.

In the following sections the city generation model is introduced and evaluated for both truncated and non-truncated rank-order distribution of axial line length. In order to discriminate exponents of the truncated and non-truncated distributions, hereafter, *z*-alpha refers to alpha exponent for the non-truncated distribution that includes all axial lines.

[5] The city with the smallest number of axial lines in Carvalho and Penn (2004) is Winchester in the UK, which has 616 axial lines.

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4. City generation model

The model used for generating the pattern of rank-order distribution of axial line length in this paper is illustrated by Rezayan et al. (2008). The model is originally developed through generation of archetype global urban patterns using a minimum set of simple[6] spatial rules interact at a local scale. The rule-set is considered to define the spatial information content of the urban patterns i.e. the minimum spatial information required to generate the urban patterns. Beside the pattern studied in this paper, generation of the small-world network (Rezayan et al., 2008) and fractals (Rezayan et al., 2010) in the model are studied too.

The model considers a city as an emergent complex phenomenon composed of free/open and closed spaces. It adopts the hypothesis that an aggregation of buildings optimally located in space by people generates urban patterns. The model does not necessarily consider the movement of vehicles defining its rule-set. These considerations depict an unplanned organic pedestrian small city which its morphological structure (connection of free and closed spaces) is not pre-defined. The structure is generated as the city grows through a bottom-up process of interaction of some simple rules with the environment at a local scale. It is expected that the results of the model follow the specifications described for small cities in Section 3.

The model used here has a set of seven simple spatial rules: (1) Distance-to-Center-of-Gravity, (2) Distance-to-Road, (3) Free-Space, (4) Adjoining-Free-Space, (5) Access-Space, (6) Adjoining-Access-Space, and (7) Sun-Position. The rule-set is defined through an iterative process of adding a simple rule and evaluating its influences on generation of the expected urban patterns. The rule is dropped if it has degenerating effects. The process of adding new rules is stopped when stable urban patterns are generated according to the specifications of real urban patterns. It results in a minimum rule-set that generates the expected urban patterns. This approach is also used by Hillier (1989) generating the beady ring pattern. He defined two rules which are admitted in the model used in this paper (rules #3 and #4).

The mentioned emergent definition of a city and the bottom-up process adopted by the model used in this paper, which indicate that the model is expected to simulate unplanned organic pedestrian small cities, discriminate it from models defined for simulation of contemporary planned cities. These models consist of a series of processes (like street extraction process, block extraction process, and parcel extraction process) that are connected to each other by the plans and regulations defined for the city being simulated (Epstein and Axtell, 1996; Parish and Müller, 2001; Lechner et al., 2004; Koenig and Bauriedel, 2009).

Parish and Müller (2001) use a pre-defined population density map to extract the main roads connecting the populated points and then use an L-System[7] to generate the streets. The blocks are defined intersecting buffers of the streets and the parcels are extracted in each block. These processes are carried out separately according to the planning rules defined for the city like plans made for the length, slope, or angle of the streets or those defined for the area, shape, and usage of the blocks and the parcels. Similarly, Lechner et al. (2004) generate the structure of the streets separately before locating the parcels. They, however, run their street extraction process focally (in a neighborhood) as the generation process proceeds. They also consider land-uses (like residential, commercial, and recreational) and define a planned process for change of the land-uses as the city ages. Koenig and Bauriedel (2009) propose a combination of bottom-up generation and top-down control. They control the permeability level of the space and the density of the settlements. Epstein and Axtell (1996) introduce Sugarscape concept and develop emergent models for simulation of wars, trades, and social interactions in city systems.[8]

These models simulate the effects of specific plans and regulations defined for a city. Stiny et al. (2008) address more of such models.

As mentioned, Hillier (1989) follows an emergent approach for generating the beady ring pattern. Ingram et al. (1996) uses the rules and the simulation process defined by Hillier (1989) to generate the passages in their model. However, they firstly extract the structure of the city as defined by Lynch (1960). The structure is a connection of paths, edges, districts, nodes, and landmarks that defines the simulation process. They use the model to evaluate the legibility of the extracted structure (Lynch, 1960). The model is used to generate test cases in which multiple path findings are executed from different start locations in the space in order to reach a target location. It is evaluated how the path finding process is improved as an avatar become more familiar with the environment.

The model used in this paper is implemented in a cellular/raster environment. Each cell is empty or represents a building, a free space, or a road. The free spaces are emergent entities. They are created and organized during the generation of a city. Roads, however, are pre-existing entities. They are also considered as free spaces. Free spaces form the passages between the buildings. Passages are processed to extract axial lines.

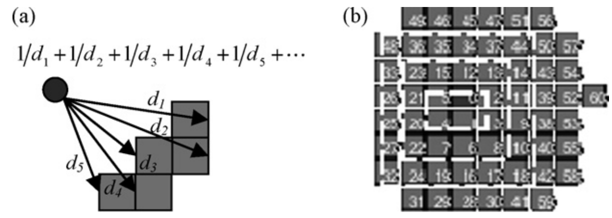


Fig. 5. (a) Schematic representation of the Distance-to-Center-of-Gravity rule. It values each location of space as the sum of its inverse distances to existing buildings as shown by gray squares (Eq. (3)). (b) This rule creates compact layer-by-layer patterns around the seed(s) (the dark square at center) placed in the environment by the user (Rezayan et al., 2008).

4.1. Simulation rules

The rules in the model are briefly described in this section. Detailed descriptions are provided in Rezayan et al. (2008).

4.1.1. Distance-to-Center-of-Gravity

The Distance-to-Center-of-Gravity rule is one of the global rules defined in the model (Rezayan et al., 2008). It asserts Tobler's Law (Tobler, 1979) that people prefer to live near each other. The rule formulates that the sum of the inverse distances from a location to all the buildings that already exist in the environment affects the value of the location for adding a new building (Eq. (3); Fig. 5a). This rule depicts distance to the center of gravity of the buildings.

$$\text{Value-of(aLocation)} \approx \sum_{\text{aBuilding}}^{\text{Buildings}} \frac{1}{\text{Distance(aLocation, aBuilding)}} \quad (3)$$

This rule aggregates the buildings, defining the feasible locations with maximum value among the immediate neighbors of the pre-existing buildings in the environment. It forms a layer-by-layer pattern (Fig. 5b).

[6] The rules are simple as their presence in the process of the city generation is considered obvious.

[7] Lindenmayer System.

[8] A group of cities interact with each others.

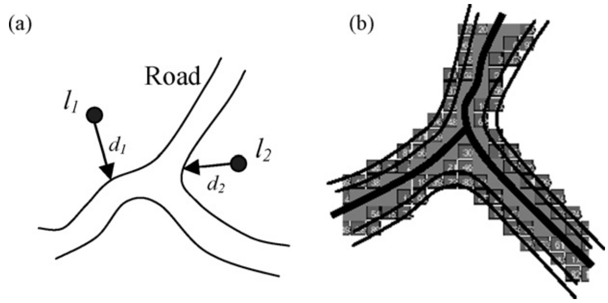


Fig. 6. (a) Schematic representation of the Distance-to-Road rule. It values each location based on the inverse shortest distance to roads (Eq. (4)). (b) This rule creates layer-by-layer patterns defined by the parallel lines around the roads (Rezayan et al., 2008).

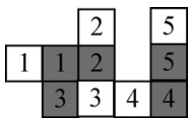


Fig. 7. Schematic representation of the Free-Space rule (Rezayan et al., 2008). The gray squares show buildings and the white squares show the free spaces. Numbers in the squares show the order of locating the buildings and free spaces. Each building and its assigned free space have the same number.

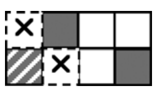


Fig. 8. Schematic representation of the Adjoining-Free-Space rule (Rezayan et al., 2008). Feasible locations for the free spaces that can be assigned to the new building (the hatched square) are marked with a cross.

4.1.2. Distance-to-Road

The Distance-to-Road rule states that distance to roads affects the value of the location for adding a new building (Fig. 6a) (Rezayan et al., 2008). It maximizes the inverse distance of a location to the roads (Eq. (3)).

$$\text{Value-of(aLocation)} \approx \max\left(\bigcup_{\text{aBuilding}}^{\text{Buildings}} \frac{1}{\text{Distance(aLocation, aBuilding)}}\right) \quad (4)$$

4.1.3. Free-Space

Hillier (1989) introduced the Free-Space rule to allow each building to have an exclusive free space attached to it (Fig. 7). This free space acts as a front yard. It enables people to go in and out of a building. The Free-Space rule is the basis of other local rules.

4.1.4. Adjoining-Free-Space

Hillier (1989) introduced the Adjoining-Free-Space rule, which forces a free space to adjoin other free spaces, if possible (Fig. 8). The emergence of terraced buildings is the main effect of this rule (Rezayan et al., 2008).

4.1.5. Access-Space

The Access-Space rule states that a road can provide its neighbor buildings with their required free spaces (Fig. 9). The neighbors share the road as their free spaces and their entrances face toward the road.

4.1.6. Adjoining-Access-Space

The Adjoining-Access-Space rule alters the Access-Space rule, stating that the buildings located near a road tend to face toward the road. The buildings instead use the gap between themselves and the road as their free spaces (Fig. 10).

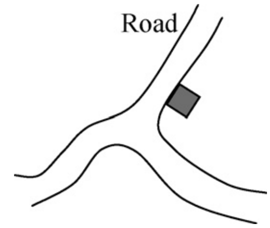


Fig. 9. Schematic representation of the Access-Space rule. (Rezayan et al., 2008). The building is shown as a gray square attached to the road.

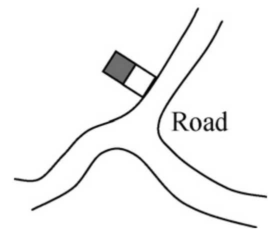


Fig. 10. Schematic representation of the Adjoining-Access-Space rule. (Rezayan et al., 2008). The building in gray is attached to the road by its free space shown by a white square.

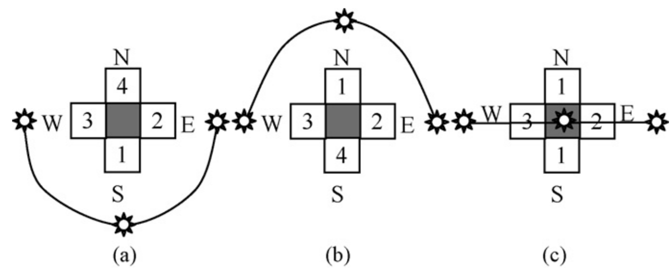


Fig. 11. Schematic representation of the Sun-Position rule (Rezayan et al., 2008). The numbers show the priority of the free space assignment to the building (gray square) in the center of the figure. The characters define South (S), East (E), West (W), and North (N) directions. (a) The northern hemisphere, (b) the southern hemisphere, (c) near equator.

4.1.7. Sun-Position

The Sun-Position rule holds that each building tends to attain the maximum duration of sunlight in their attached free space. Then the free space of a building is assigned based on the cardinal directions. Directions are prioritized as south, east, west, and north (Lynch, 1981) (Fig. 11a). This prioritization represents the regions in the northern hemisphere. Fig. 11b and c show the priority for the regions in southern hemisphere and the priority for the regions near equator respectively.

Considering that none of the rules defines the geographical cardinal directions, the effects of the Sun-Position rule would be the same for the northern and southern hemisphere locations. The near the equator case is not studied in this paper.

4.2. Simulation process

Each simulation of the model starts by adding one or more buildings as seeds, and perhaps some roads in the environment. These entities can be imported into the model from a map or drawn and edited by the operator.

At each step of the simulation, one building and its required free space is added to the environment. The locations of the building and its free space are defined by applying the rules (Section 4.1) to

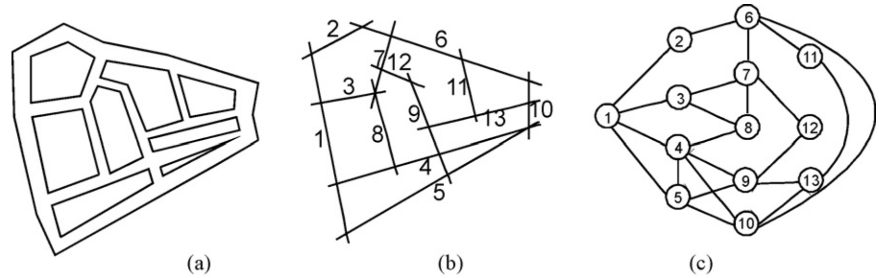


Fig. 12. Axial representation of space (Jiang and Claramunt, 2000). (a) A scheme of a linear space in which dense settlement forced relatively linear passages along the building. (b) The axial lines derived from the scheme shown in (a). (c) The connectivity graph of the axial lines in which the nodes are the axial lines and the links represent the connections between axial lines.

the existing spatial arrangement of the entities (roads, buildings, and free spaces) in the environment. Accumulation of the buildings and their free spaces generates the expected city environment and patterns.

At first, location of the building (the closed space) is defined using Distance-to-Center-of-Gravity and Distance-to-Road rules, which interact with the environment at a global scale. One of the cells with the maximum normalized value is selected randomly.[9]

Location of the required free space for the selected building is defined by applying the rest of the rules: (1) Free-Space, (2) Adjoining-Free-Space, (3) Access-Space, (4) Adjoining-Access-Space, and (5) Sun-Position. These rules interact with the environment at a local scale. They are applied in the order enumerated. The process of locating the free space stops when one possible cell is remained. If more than one possible cell is remained, after the rules are applied, one of the cells is selected randomly. The Sun-Position rule used (Fig. 11a and b), however, discards the random selection of free spaces and ensures that the possible locations for a free space are always reduced into one cell.

The Sun-Position rule interacts at a local scale, although it has global/similar effect on all the locations. Avoiding dominance of the global effect of the Sun-Position rule, it is applied as the last rule, if more than one possible cell is remained.

4.3. Axial representation in the model

Axial lines are defined in a space that provides axial representation (Jiang and Claramunt, 2000). In such a space the built environment is relatively dense, so the free spaces are aligned in one orientation at most of the points (Fig. 12).

The city generation model used and its rule-set generate a compact and relatively linear structure. It provides the axial representation required to define and extract axial lines. The compactness in the model is mainly due to the application of the Distance-to-Center-of-Gravity rule and the Distance-to-Road rule that create compact layer-by-layer patterns. The rules that order the free spaces, especially the Sun-Position rule and the Adjoining-Free-Space rule, increase the linearity of the model. The cellular environment used for the implementation of the model exerts more compactness and linearity in the simulations (Rezayan et al., 2008).

4.4. Automatic extraction of axial lines

Jiang and Liu (in press) and Turner et al. (2005) review the algorithms proposed for automatic extraction of axial lines. Most of the algorithms work in vector space. The city generation model here, however, is simulated in a cellular/raster space.

[9] The selection is based on a pseudo random process which ensures that similar inputs always result in similar outputs in the model.

Carvalho and Batty (2005) propose a method for automatic extraction of axial lines in raster space. Their algorithm derives the alignment of axial lines but does not define the start and end points. As length of an axial line is the basic property used in the evaluations, their algorithm is therefore not applied.

Among the algorithms provided for automatic extraction of the axial lines in vector space (Jiang and Liu, in press; Turner et al., 2005), the algorithm introduced by Jiang and Liu (in press) is adopted here. The adopted algorithm uses a global loop-search method and denoted as "Algorithm II" (Jiang and Liu, in press).

The simulation process of the model described earlier in Section 4, simplifies space into a cellular arrangement of spatial elements (free spaces, roads, and buildings). It also causes the emergence of narrow (mostly one-cell wide) passages. Then the following adaptations are carried out using "Algorithm II" defined by Jiang and Liu (in press) in raster space:

1. A ray is defined between the center of two cells marked as passage and not obstructed by other non-passage cells. Then, each ray is defined as a list of connected cells.
2. Two rays are considered to be connected if they cross over or touch each other.
3. The ray-set is formed by extracting all possible rays between the cells marked as passages along the ridges as defined by Carvalho and Batty (2005).
4. Overlap of the convex areas that a ray intersects is extracted as the bucket of the ray (Jiang and Liu, in press). The bucket area is bounded by the cells perpendicular to the ridges at the boundary points of the ray.

The complexity of the algorithm is $O(n^2)$ where n denotes the number of the cells that represent the free spaces. It is proposed to minimize the number of the cells while preserving the geometrical and topological structure of the free spaces. No approach is provided here to define the optimum dimension for rasterizing the space and a trial and error approach is used.

Fig. 13. Overlay of the layout of the town of Gassin in France (Hillier, 1989) and the medial axes extracted along the ridges using the algorithm described in Carvalho and Batty (2005). It is a 100 × 50 cellular representation of the city layout. The black cells represent the passages and the lighter cells represent the medial axes of the passages.

Table 1
Properties of the alpha exponent of the axial map for the town of Gassin in France shown in Fig. 16. The lengths are measured in cells.

Type	Count of axial lines	Minimum length	Maximum length	Mean length	Median length	Power law exponent
Truncated (alpha)	11	26	75	42	33	1.824
Non-truncated (*z*-alpha)	42	4	75	19	13	1.327

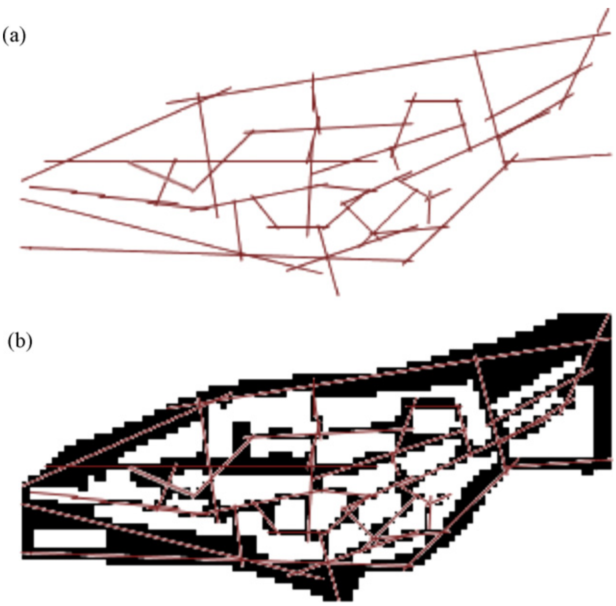
(a)
(b)


Fig. 14. (a) Axial map of the town of Gassin in France in raster space. There are 43 axial lines. Axial lines one-cell apart are still attached as their cells touch each other. (b) Overlay of the layout of Gassin and its extracted axial lines in raster space.

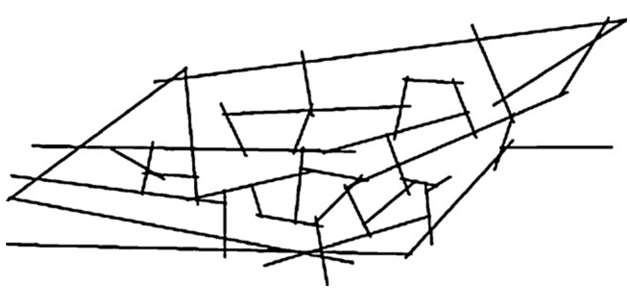


Fig. 15. Axial map of town of Gassin in France extracted by Jiang and Liu (in press) in vector space. It has 39 axial lines.

To check the algorithm, it is applied for the town of Gassin in France,[10] which is also used by Jiang and Liu (in press). The results are compared to Jiang and Liu (in press). The layout of the city is rasterized into 100 × 50 cells (Fig. 13).

The main artifact of rasterizing the space is the possible change in the direction of short axial lines (Fig. 14a and Fig. 15). The area of free spaces decreases using a larger cell size for rasterizing the space. It decreases the number of axial lines.

Fig. 16 shows the rank-order distribution of axial line length for Gassin. Properties of the truncated and the non-truncated distributions are presented in Table 1. In both cases the distributions lean toward short axial lines, although a few long axial lines exist (Fig. 17). The *z*-alpha is 1.327. It shows that the distribution leans toward short axial lines.

[10] The town of Gassin in France is a typical case study for evaluation of the algorithms related to axial lines, introduced by Hillier (1989). It is a pedestrian city located at south east of France. It is 25 km^2 with about 3000 population (Wikipedia).

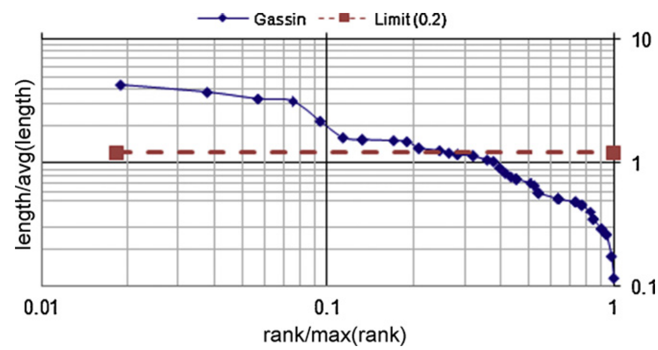


Fig. 16. Distribution of length of 43 axial lines extracted for the town of Gassin in France in raster space. The axes have a logarithmic scale.

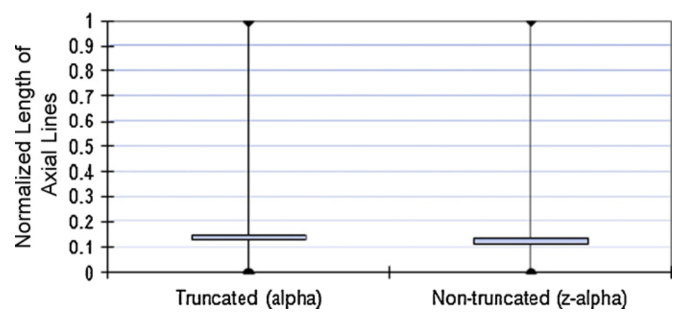


Fig. 17. Range graph of Table 1. The vertical lines show the range of minimum and maximum length of axial lines normalized to 0–1. The horizontal lines show the median length normalized to 0–1. The median lengths for both truncated and non-truncated distributions lean toward short axial lines. The normalized median lengths are equal to 0.15 for the truncated distribution and 0.13 for the non-truncated distribution. The *z*-alpha of Gassin is 1.327 (Table 1) which shows that the distribution leans toward short axial lines.

5. Discussion

The discussion is focused on how local and global levels of space interact in the city generation model described in Section 4. It is expected that the local rules have considerable effects on the results and could perturb the dominance of the global spatial structure, and also cause global stability in the emergence of the rank-order distribution of axial line length. Emergence of the pattern applying the global rules is discussed first (Section 5.1), followed by the effects of the local rules (Section 5.2), evaluating eight possible rule combinations.

As mentioned, both the truncated and non-truncated rank-order distribution of axial line length are evaluated for the simulations carried out. Emergence of the power law is investigated for each simulation according to the specifications defined for small cities in Section 3. Properties of the alpha and *z*-alpha exponents are presented and their range graphs are drawn as the one shown for Gassin in Fig. 17.

The general properties of the simulations carried out are as follows:

1. The lengths are measured in cells.
2. No uncertainty is exerted in generation, extraction, length measurement, and counting of the axial lines that form our observations. The decimals used to show the values of alpha

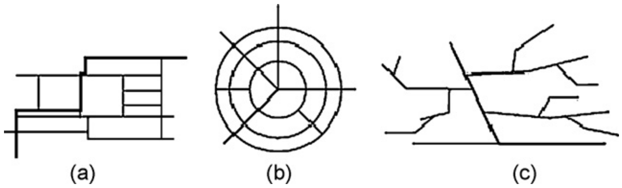


Fig. 18. Schematic representation of three frequent road structures (Parish and Muller 2001). (a) A grid structure, which is typical in modern and preplanned areas of cities. It is used to facilitate the traffic flow of cars. (b) A circular structure can be found in cities that form around specific centers of gravity like a mosque, church, or temple. (c) An organic structure is the basic framework of un-preplanned cities. The organic structure of roads is used in simulations here.

and *z*-alpha exponents do not represent uncertainty or level of definiteness in the model and the observations. The number of decimals, (which is three here) is the minimum number selected to distinguish the simulations based on their alpha and *z*-alpha exponents.

3. NetLogo 3.1.2 (http://ccl.northwestern.edu/netlogo/) is used to simulate the model and extract the axial lines (Wilensky, 2006).

5.1. Effects of the global rules

The global rules of the model are (1) Distance-to-Center-of-Gravity and (2) Distance-to-Road. Their effects are evaluated here.

5.1.1. Distance-to-Center-of-Gravity

As shown in Fig. 5b, the Distance-to-Center-of-Gravity rule forms a compact layer-by-layer pattern. It provides the basis for the generation of an axial representation of space as described by Jiang and Claramunt (2000), shown in Fig. 12.

5.1.2. Distance-to-Road

The Distance-to-Road rule causes the model to grow layer-by-layer along the roads (Fig. 6b). It behaves like the Distance-to-Center-of-Gravity rule in compacting space and generating the axial representation (Jiang and Claramunt, 2000) (Fig. 12).

As mentioned in Section 4, roads are part of the initial structure of the model. The operator enters/draws roads into the model as a pre-existent entity.

The shape of the roads affects the structure of axial lines and the distribution of their lengths. Fig. 18 shows the schemes of three frequent road structures (Parish and Müller, 2001).

Figs. 19–21 show the axial maps of the three road structures. The axial maps are extracted from a 100 × 50 cellular representation of the schemes shown in Fig. 18. Fig. 22 shows their rank-order distribution of axial line length. Tables 2 and 3 present properties of the alpha and *z*-alpha exponents for the three axial maps. Figs. 23 and 24 show their range graphs.

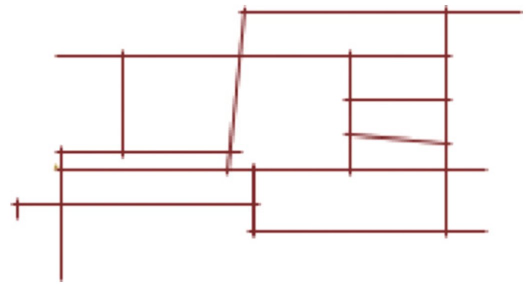


Fig. 19. Axial map of the grid road structure (Fig. 18a). There are 15 axial lines which are mainly of medium length.

Fig. 20. Axial map of the circular road structure (Fig. 18b). There are 43 axial lines consisting of 4 long and 39 short axial lines.

Fig. 21. Axial map of the organic road structure (Fig. 18c). There are 23 axial lines ranging from long to short.

The *z*-alpha exponents of the grid and the organic structure of roads (Table 3) are near 1. It represents their hierarchical structure and their growth stability (described in Section 4.1). The axial lines of the grid structure, however, are medium to long. Then dominance of the local spatial structure is higher in the organic road structure and it has more depth (short axial lines) and stability.

The circular road structure, however, is not hierarchical. It has four very long axial lines (like wheel spokes), which are about 26 cells long, while the rest of the 39 axial lines are under 14 cells long. This caused instability in alpha (14.748) and *z*-alpha (2.77) exponents (Tables 2 and 3) of the circular road structure.

Table 2
Properties of the alpha exponent of axial maps for Fig. 22.

Road structure	Count of axial lines	Minimum length	Maximum length	Mean length	Median length	Alpha exponent
Grid	5	27	49	36	32	2.426
Circular	4	25	28	26.375	26	14.748
Organic	6	22	60	32	25	1.703

Table 3
Properties of the *z*-alpha exponent of axial maps for Fig. 22.

Road structure	Count of axial lines	Minimum length	Maximum length	Mean length	Median length	*z*-alpha exponent
Grid	15	2	49	21.466	19	1.175
Circular	43	6	28	11.686	10.5	2.77
Organic	23	3	60	15.587	10.5	1.26

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Table 4
Properties of the z-alpha exponent of axial maps for Fig. 25.

Branch no.	Count of axial lines	Minimum length	Maximum length	Mean length	Median length	z-Alpha exponent
A	8	4	21	9.562	9	1.425
B	6	8	24	14.666	13	1.592
C	8	3	24	11.5	10.45	1.17

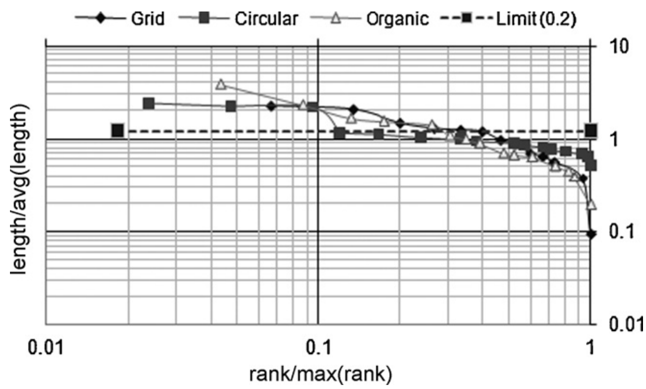

Fig. 22. Rank-order distribution of axial line length for the three typical road structures (Figs. 19–21). The axes have a logarithmic scale.

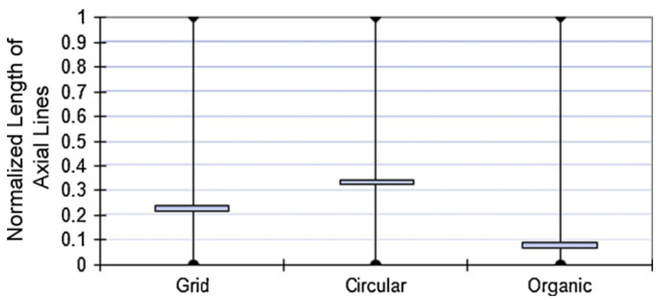

Fig. 23. Range graph of Table 2.

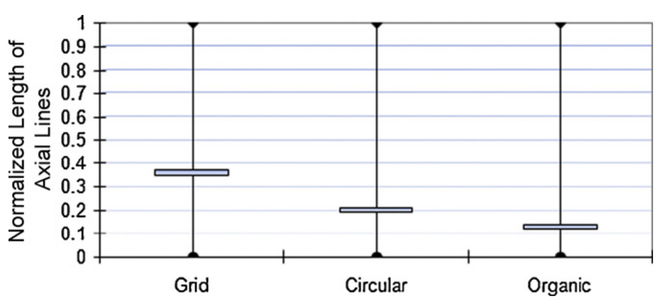

Fig. 24. Range graph of Table 3.

Considering that the city generation model developed to grow organic, pedestrian, and unplanned small cities, roads entered into the model will have an organic structure. The organic road structure shown in Fig. 18c represents two major roads (the straight line from top to bottom and the one from left to right) which branch out at three locations. The intended cities emerge on these kinds of road branches and, more likely, at intersections.

Axial maps of the three branches (Fig. 18c), their z-alpha properties, and their range graphs are shown in Figs. 25 and 26, Table 4. The axial maps are hierarchical. Similarity of the z-alpha exponent properties of the axial maps (which are all near 1) (Table 4), represents the self-similarity that exists in the organic road structure.

The three branches, as samples of the organic road structure, are used in the simulations carried out in the following sections. Their effects are normalized through repeated simulations.

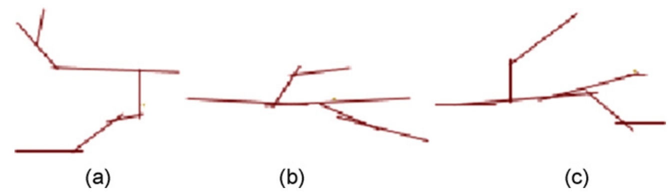

Fig. 25. Axial maps of the three branches of the organic road structure shown in Fig. 21.

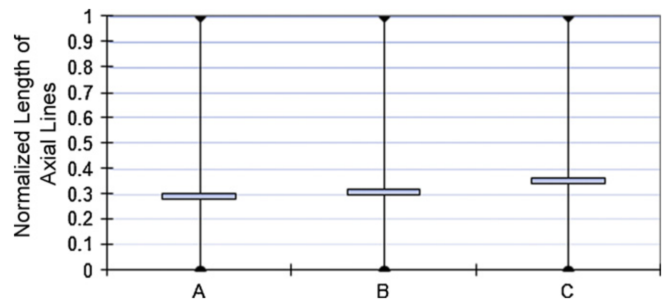

Fig. 26. Range graph of Table 4.

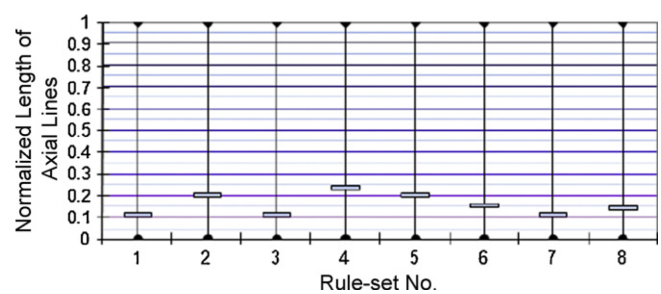

Fig. 27. Range graph of Table 5.

5.2. Effects of the local rules

The local rules are (1) Free-Space, (2) Sun-Position, (3) Adjoining-Free-Space, (4) Access-Space, and (5) Adjoining-Access-Space. Properties of the alpha and z-alpha exponents for eight rule-sets that represent possible combinations of the local and global rules are shown in Tables 5 and 6.

The results for each rule-set are derived from 10 iterations using different roads (the three branches in Fig. 25) and different building seeds (single, multiple, attached, and detached). The rule-sets are ordered to show the increase in dominance of the local spatial structure and the stability of the generated pattern when new rules are added to the model (Figs. 27 and 28).

The Free-Space rule is applied in all of the eight rule-sets. It does not generate any specific pattern by itself (Fig. 7) and its influences are therefore studied with other rules.

As mentioned in Section 4.2 the Sun-Position rule has global effect but it is applied at a local scale (Fig. 11). This rule exerts a higher level of order in the space than the other local rules. It causes emergence of longer axial lines. Studying the effect of the Sun-Position rule, simulations are evaluated for both the Sun-Position

Table 5
Properties of the alpha exponent of axial maps for eight rule-sets representing possible combinations of the rules defined in the model. Each rule-set is repeated 10 times with different arrangements of road structures and building seeds. The lengths are measured in cells.

Rule-set no.	Rules							Properties of the alpha exponent of axial maps						
	Distance-to-Center-of-Gravity rule	Distance-to-Road rule	Free-Space rule	Sun-Position	Adjoining-Free-Space rule	Access-Space rule	Adjoining-Access-Space rule	avg (alpha)	stddev (alpha)	min (length)	max (length)	avg (length)	avg(median (length))	stddev (median(length))
1	*		*		*			3.171	0.602	8.5	30	12.386	11	1.414
2	*		*	*	*			4.841	1.785	11	30	15.782	14.9	0.742
3	*	*	*		*			2.057	0.253	10	43	16.988	13.75	1.639
4	*	*	*	*	*			2.155	0.136	11	40	20.384	17.958	2.272
5	*	*	*		*	*		2.233	0.144	10	40	18.331	16.125	3.474
6	*	*	*	*	*	*		2.394	0.327	10.5	46	18.354	16.042	1.631
7	*	*	*		*	*	*	2.201	0.173	9.5	49	17.128	14	1.924
8	*	*	*	*	*	*	*	2.684	0.316	11	49	18.463	16.542	2.076

Table 6
Properties of the *z*-alpha exponent of axial maps for eight rule-sets representing possible combinations of the rules defined in the model. Each rule-set is repeated 10 times with different arrangements of road structures and building seeds. The lengths are measured in cells.

Rule-set no.	Rules							Properties of the *z*-alpha exponent of axial maps						
	Distance-to-Center-of-Gravity rule	Distance-to-Road rule	Free-Space rule	Sun-Position	Adjoining-Free-Space rule	Access-Space rule	Adjoining-Access-Space rule	avg (*z*-alpha)	stddev (*z*-alpha)	min (length)	max (length)	avg (length)	avg (median (length))	stddev (median (length))
1	*		*		*			2.04	0.1	2	30	7.07	6	0.90
2	*		*	*	*			1.97	0.1	2	30	9.19	8.25	1.06
3	*	*	*		*			1.67	0.07	2	43	8.46	7	0.98
4	*	*	*	*	*			1.45	0.16	2	40	10.4	8.92	1.8
5	*	*	*		*	*		1.46	0.1	2	40	9.04	7	0.84
6	*	*	*	*	*	*		1.56	0.09	2	46	9.36	7.67	0.52
7	*	*	*		*	*	*	1.52	0.13	2	49	8.53	6.67	0.88
8	*	*	*	*	*	*	*	1.45	0.11	2	49	9.53	7.38	0.8

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Fig. 28. Range graph of Table 6.

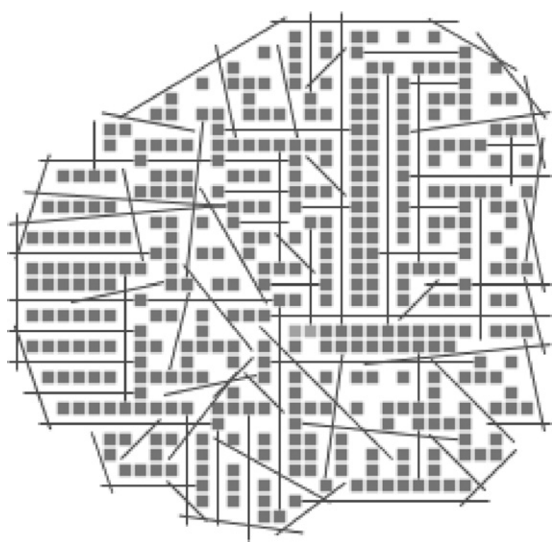

Fig. 29. A sample simulation using rule-set #1.

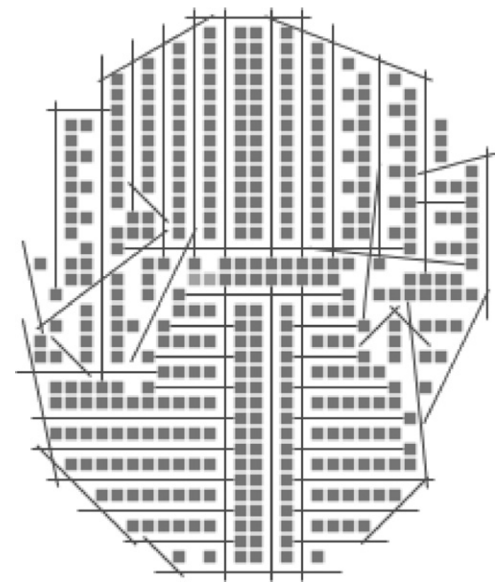

Fig. 30. A sample simulation using rule-set #2.

Fig. 31. A sample simulation using rule-set #3.

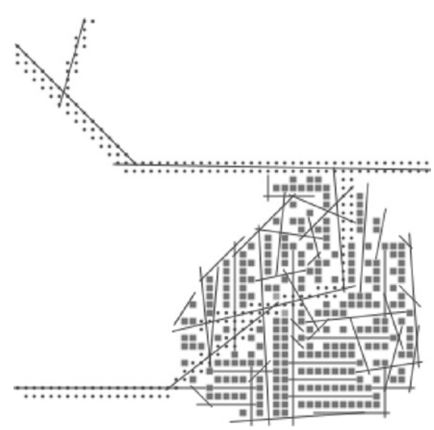

Fig. 32. A sample simulation using rule-set #4.

rule being off (rule-sets #1, #3, #5, and #7) and on (rule-sets #2, #4, #6, and #8). Each odd rule-set forms a couple with its next even rule-set (Couples: [#1, #2], [#3, #4], [#5, #6], and [#7, #8]). It is expected that for each couple the distribution of the odd rule-set leans toward short axial lines more than the even rule-set (Figs. 27 and 28).

Rule-set #8 includes all the rules. Its alpha exponent equals 2.684 (with stddev = 0.316), which is higher than the other rule-sets, except rule-sets #1 and #2. It shows that the influence of the local rules is higher in rule-set #8. The *z*-alpha exponent of rule-set #8 (1.45) is also among the lowest. It represents a stronger hierarchical structure in space that is dominated by local rules. The stability of the *z*-alpha exponent for rule-set #8 (defined by its standard-deviation which is equal to 0.11) is also high.

Properties of each rule-set and the effects of each rule are described in the following sections. A sample of the simulations for each rule-set is presented in Figs. 29–36 and Tables 7–14. The buildings are shown in Figs. 29–36 as gray squares, the roads as dots, and axial lines as gray lines. The free spaces emerged in the model, which are in white, can be traced along the axial lines.

5.2.1. *Adjoining-Free-Space*

The emergence of terraced buildings is the main effect of the Adjoining-Free-Space rule (Rezayan et al., 2008), and it can cause the emergence of long axial lines. This effect is shown in rule-sets #1 and #2 (Figs. 29–32) where their distributions lean toward long axial lines more than the other rule-sets.

5.2.2. *Access-Space*

The Access-Space rule (Fig. 9) causes roads to behave as passable edges, even when rules like the Distance-to-Road rule are not used. This rule, however, creates more life-like patterns around the roads than using the Distance-to-Road rule alone. It also compacts the free spaces along the roads.

Table 7
Properties of the alpha and z-alpha exponents in Fig. 29.

Type	Count of axial lines	Minimum length	Maximum length	Mean length	Median length	Power law exponent
Truncated (alpha)	11	26	75	42	33	1.824
Non-truncated (z-alpha)	89	2	20	19	7	2.027

Table 8
Properties of the alpha and z-alpha exponents in Fig. 30.

Type	Count of axial lines	Minimum length	Maximum length	Mean length	Median length	Power law exponent
Truncated (alpha)	15	13	21	16.133	16	5.327
Non-truncated (z-alpha)	56	3	21	9.875	9.75	2.053

Table 9
Properties of the alpha and z-alpha exponents in Fig. 31.

Type	Count of axial lines	Minimum length	Maximum length	Mean length	Median length	Power law exponent
Truncated (alpha)	16	10.5	39	16.656	14	2.348
Non-truncated (z-alpha)	87	2	39	8.22	7	1.740

Table 10
Properties of the alpha and z-alpha exponents in Fig. 32.

Type	Count of axial lines	Minimum length	Maximum length	Mean length	Median length	Power law exponent
Truncated (alpha)	10	14	38	22.5	20.245	2.380
Non-truncated (z-alpha)	54	2	38	9.787	8	1.450

Table 11
Properties of the alpha and z-alpha exponents in Fig. 33.

Type	Count of axial lines	Minimum length	Maximum length	Mean length	Median length	Power law exponent
Truncated (alpha)	9	14	39	22.777	22	2.188
Non-truncated (z-alpha)	36	2	39	10.435	7.5	1.351

Table 12
Properties of the alpha and z-alpha exponents in Fig. 34.

Type	Count of axial lines	Minimum length	Maximum length	Mean length	Median length	Power law exponent
Truncated (alpha)	14	13	40	20.714	17.75	2.306
Non-truncated (z-alpha)	60	2	40	10.2	8	1.480

The application of the Access-Space rule in rule-sets #5, #6, #7, and #8 perturbs the structure of long axial lines generated using rule-sets #1, #2, #3, and #4. It causes the emergence of more short axial lines (Figs. 27 and 28). The perturbations are increased near the roads. Some of the emerged short axial lines connect the road sides, like alleys (Figs. 33 and 34).

Application of the Sun-Position rule in rule-set #6 reduced the number of short axial lines (Figs. 27 and 28). The change in the z-alpha exponent between rule-sets #5 and #6 is 0.1, which is lower than the previous couples. It shows that the Access-Space rule made more significant and more stable effects in the results.

5.2.3. Adjoining-Access-Space

Applying the Adjoining-Access-Space rule (Fig. 10), the free spaces of buildings are located around the roads, widening the roads and forming passages that look like sidewalks (Figs. 11, 34 and 36). In comparison with the Access-Space rule, the Adjoining-Access-Space rule creates less compaction of buildings around the roads. It, however, allows the free spaces to connect more freely to the road. This causes the emergence of more alley-like short axial lines connecting roadsides (Figs. 34 and 36). A few of this kind of axial lines emerge, using the Access-Space rule, as shown in Figs. 33 and 34.

The application of the Sun-Position rule in rule-set #8 (Fig. 36) reduced the short axial lines. The change in the z-alpha exponent between rule-sets #7 and #8 is 0.07, which is among the lowest changes in the couples (Table 6; Fig. 28). It shows that application of the Adjoining-Access-Space rule provides more stability and subtlety in the generated pattern.

Table 13
Properties of the alpha and z-alpha exponents in Fig. 35.

Type	Count of axial lines	Minimum length	Maximum length	Mean length	Median length	Power law exponent
Truncated (alpha)	17	11	42	19.205	16.5	2.117
Non-truncated (z-alpha)	76	2	42	8.480	6	1.324

Table 14
Properties of the alpha and z-alpha exponents in Fig. 36.

Type	Count of axial lines	Minimum length	Maximum length	Mean length	Median length	Power law exponent
Truncated (alpha)	24	13	45	20.77	19	2.613
Non-truncated (z-alpha)	85	2	45	10.223	7	1.314

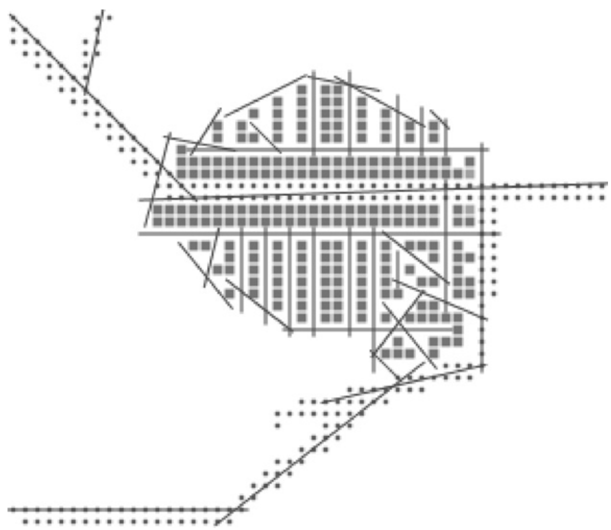

Fig. 33. A sample simulation using rule-set #5.

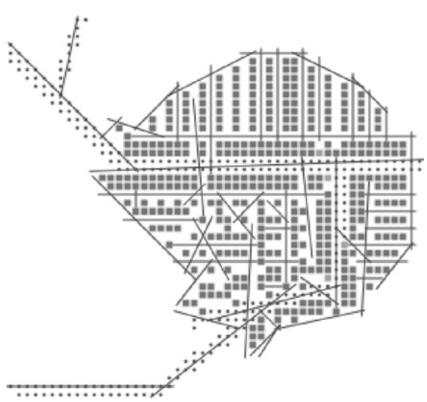

Fig. 34. A sample simulation using rule-set #6.

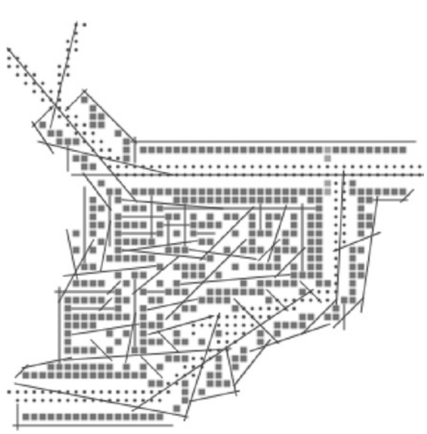

Fig. 35. A sample simulation using rule-set #7.

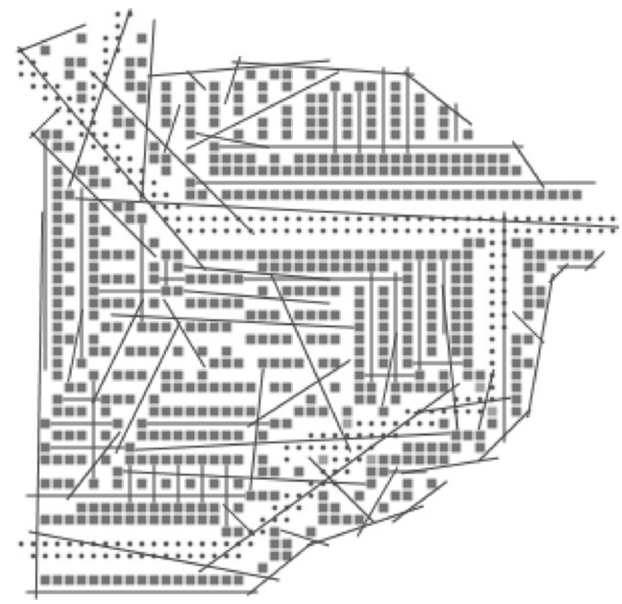

Fig. 36. A sample simulation using rule-set #8.

6. Conclusions

This paper has shown that the city generation model proposed by Rezayan et al. (2008) and its minimum set of its seven simple spatial rules provide the required compactness and linearity to form axial representation in space. The results follow the pattern illustrated by Carvalho and Penn (2004) with its alpha exponent equal to 2.684 (with stddev = 0.316) that stands near the stable region they defined (Fig. 2). The z-alpha exponent of the generated pattern is equal to 1.45 (with stddev = 0.11), which represents the significant role of the local rules, causing the emergence of more short axial lines.

The discussions admit that the global rules lead cities toward the generation of longer axial lines. It represents the existence of a rather steady growth of free spaces in the simulated environments. The local rules, however, break this steady growth in favor of faster changes at a local scale. The model still allows for slow and steady growth at a global scale, however. The effects cause the emergence of a hierarchical environment. Such an environment supports the generation of life-like structures like cities.

The pattern is generated as the result of local interactions of the seven simple spatial global (2 rules) and local rules (5 rules). The effects of the Access-Space and the Adjoining-Access-Space local rules are more significant in altering the global spatial structure, causing the emergence of alley-like passages around the roads.

The adopted algorithm for the automatic extraction of axial lines in raster space described in Section 4.3 should be studied for the effects of rasterization dimensions. These effects would be more visible in the environments with wide passages. Considering the simplicity of the simulation space used in this paper (Section 5), especially the compactness, linearity and narrowness of the passages, the current settings of the algorithm were applied here.

The city generation model presented is used as an evaluation model for the archetype global urban pattern studied here. To illustrate, the emergence of the small-world network pattern (Rezayan et al., 2008) and fractals (Rezayan et al., 2010) are studied too. The model simulates unplanned organic pedestrian small cities. It does not simulate any specific urban plans and regulations. Considering these, possible applications of the model are as follow:

1. More archetype global urban patterns can be studied. It may need to add new rules their effects should be studied for all the patterns the rule-set generates.
2. It can be used to simulate test cases for algorithms that rely on existence of the urban patterns the model generates.
3. The rule-set explains some basic spatial aspects of pedestrian environments. These can be taken into account fostering pedestrian environments in our cities.

The model can also provide a basis to start simulation of a specific group of cities (like the cities which are located on tidal flats or highlands) which follows by admitting their particular patterns. These patterns are studied and the rules caused their generation are defined and added to the rule-set. The new rule-set is used for simulating the city and the effects of the rules are evaluated.

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Glossary

Axial Line: The longest line of sight in an area that represents a convex area of free spaces.
Axial Map: The minimum set of axial lines that span all the convex spaces in an environment like a city.
Axial Representation: A relatively linear space where the built environment is dense. So, the free space is aligned in one orientation at most points.
Life-Likeness: The quality of being similar to real world phenomena, such as a real city.
Emergence: The process of generation of a complex structure at a global level because of interactions of simple substructures at a local level.
Power Law: In a distribution that follows power law, the size/frequency of the events increases as the size of the event decreases: the large events are rare. The power law distribution can be found in many natural and man-made phenomena which are grown through a bottom-up process of autonomous interactions at a local scale. These phenomena are scale-free and have hierarchical structures. They are flexible locally and stable globally as they experience a faster rate of growth at a local scale while having a steady growth at a global scale. Mathematically, the power law represents a distribution where its $\log(x)$ has a linear relationship with $\log(y)$.
Small-World Network: A class of networks where its nodes are connected by both long and short links. Each node in a small-world network can reach most of the other nodes by a small number of steps, although most of the nodes are not neighbors.