Select the appropriate map depending on context in a Hilbert Space Model (SCOP)

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Abstract. Human use of categories exhibits a prototype effect; concepts become more defined through a conversation. Modelling these gradual clarification of what a word signifies is equally important in human - computer interactions, for example in interactions about geographic concepts and the information that is needed in a given situation. We address here the simplified, but essentially realistic, question of what is meant by "map" and how the concept is refined. We apply the methods Aerts, Gabora and Rosch have described and explore how they can be integrated into practical systems.

In this paper we explore the optimal selection of a map through a conversation with the client to elucidate their intentions. The example case contains effects which are similar to the "guppy effect" that is known from literature and is a key reason to apply quantum mechanic formalism. The results are promising, and we sketch the extension to the construction of "custom made" maps from layers. This will provide users with maps that optimally reflect what map elements should be visible for use in a given context.

Keywords: Map Prediction, Geographic Concepts, Hilbert Space Model, SCOP

1 Introduction

Rosch has demonstrated prototype effects in the use of concepts by humans; the same word may have different meanings depending on the context. Montello [18] lists the influence of context as one of the most important problems for GIScience research and asks for the incorporation of models of human categories in Geographic Information Systems.

Aerts, Gabora, and Rosch have described a computational theory of prototypes based on quantum mechanical formalizations [12]. Their survey of previous efforts to compute categories showing prototype effects led them to conclude that a formalization has to deal with contextuality, concept combination, similarity,

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compatibility, and correlation [12]. It can be achieved with a formalism based on quantum mechanics.

The interactions between humans or between humans and computerized information systems is based on the exchange of words (or graphical tokens on maps) which are interpreted in the *context* of the conversation. The words used may originally have a broad meaning (comparable to the "pet" in Aerst, Gabora, and Rosch [12]); through conversation the context becomes more precise, and the categories obtain more specific values (e.g. "goldfish" or "snake").

Understanding the meaning of natural language words is important in information retrieval and database access; the use of quantum mechanics as a formalization has been discussed [23], and compared there with other fuzzy and probabilistic methods for data retrieval. The focus of the current paper is more comparable to the work by Aerts and Gabora [3]; it intends to apply their insight into the field of geographic information science [20] where, as mentioned before, context is assumed to be one of the major research challenges today. The proof of concept we present here is meant to understand the context of the user, and to determine the best response, without relying on the user to select among technical terms that assume technical knowledge on the user side. This is somewhat similar to a recommender system [17] for which others have suggested methods based on quantum mechanics. Relations can be drawn to Formal Concept Analysis [13].

It should be possible for a user to describe their situation from their point of view - as presented by other statements made - and for the system to then guess the most likely optimal response to the user request. Through the additional information, the produced contexts transform the concept initially invoked in the base state into a more specific one.

The paper is intended as a "proof of concept" of the applicability of the method in regards to a practical problem in Geographic Information Systems [16]. For a proof of concept we restrict the selection to the selection of a number of predetermined map types (e.g. street map, political map, map for hiking, ski routes). The goal is a computational model which can be incorporated into GIS software. The input for user preferences is produced by the authors by introspection; a real-use system would need data from a user group. Aerts and Gabora [11] have shown how such contextual frequencies can be obtained by questionnaires. It is likely that methods of Volunteered Geographic Information [14] could be used to obtain valid data for different user groups.

The paper is structured as follows: the next section outlines a brief survey of prototype effects, methods to deal with them, and the computational model Aerts, Gabora and Rosch propose; it concludes with an overview of the SCOP model used here. That section is mostly intended to establish the terminology used, and to make the paper self-contained. The following section discusses categories of maps, the map production process, and how maps are used to set the stage for the production of "customized maps" in a given context. Section 4 introduces the example proof of concept case and the context-dependent selection probabilities. Section 5 connects two contexts in an entangled state. The concluding section lists further application opportunities of quantum mechanics in geographic information processing, and discusses research issues necessary to overcome possible impediments to their widespread use.

2 Review of Theories and Models for Categories

Rosch and Mervis [21] studied the internal structure of categories. They hypothesised that family resemblance correlates with the prototypicality of items, and used polls to confirm their hypothesis. They concluded that categorized elements have some attributes in common with a prototype. This prototype can be seen as a reference point for a concept [22]: The instances of a concept are more or less prototypical and are ranked in a graded structure around the prototype.

They used fuzzy set theory [27] that is able to handle objects with graded boundaries. Smith and Osherson [24] demonstrated that fuzzy sets cannot completely model how humans use concepts. They asked people to rate the typicality of instances for the concepts: pet, fish, and pet-fish. It was found that a guppy is more typical for the combination pet-fish than for the constituent concepts (pet, fish).

Gabora et al. found that none of the then known theories formalize the effects of: (1) contextuality, (2) concept combination, and (3) similarity, compatibility, and correlation [12]. These three effects were analysed by Gabora [10]. She illustrates the contextual effect for a concept as shown in Figure 1. Starting with no contextual influence at time t0, concept p can collapse into all possible states p1(t1), p2(t1), p3(t1), and p4(t1). Influencing the concept by a particular context e3 the concept realizes state p3(t1). In state p3(t1) the concept can also collapse into all states p1(t2), .., p4(t2). Where context e7 influences the concept which collapses in state p7(t2).

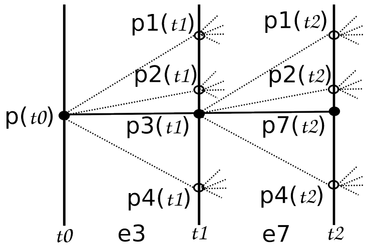

Fig. 1. Influence from contexts to concept states, see figure 11.1 [10]

As a result of this analysis, Gabora et al. [3] presented a different formalization for concepts. They called their approach state-context-property (SCOP) formalism based on quantum mechanics. They mapped elements taken from operational foundations of quantum mechanics like states, measurements, and observables to concepts, contexts, and properties of human cognition, and they chose Hilbert spaces as the foundation for the model. In Hilbert spaces all possible states for a concept can be included. An example with one concept in three states and eight contexts is shown above in Figure 1.

The model is defined as a formal model [1] $(\Sigma, \mathcal{M}, \mathcal{L}, \mu, \nu)$. The sets are:

- $\Sigma = \{p1, p2, ...\}$ representing the *states* a concept can assume
- $\mathcal{M} = \{e1, e2, ..., f1, f2, ...\}$ including *contexts* for a concept
- $\mathcal{L} = \{a1, a2, ...\}$ containing *properties* or features for a concept

The functions are:

- $\mu\,(q, e, p)$ calculates the transition probability from one state q to another state p under the influence of context e
- $\nu\,(p, a)$ weights the importance of one property a in a particular state p

If no context is applied to a concept, the state is called ground state $x_{\widehat{p}}$ [4].

(1) The entanglement that is typically found in microscopic quantum system can model combined concepts [11]. Combining two concepts with two distinct probability values into one concept creates a new probability value that cannot be split again into two probability values.

(2) The guppy effect described before can be formalized by the interference effect [7]. The Liar [2], Ellsberg and Machina [6] paradoxes can also be formalized with SCOP.

3 Characteristics of Geographic Maps

The American Heritage Dictionary provides four definitions of maps. The most suitable in the scope of this paper is as follows: "A map is a representation, usually on a plane surface, of a region of the earth ..." . As a representation, certain features or aspects of geographic entities are not taken into account, whereas others are emphasized [8]: A roadmap includes no isolines, but highlights the highway.

Smith and Mark [25] listed properties of geographical concepts (geographers use the term "feature") and found: (1) Geographic objects are tied intrinsically to geographic space and inherit many properties from it, such as topology and geometry. (2) The scale used to categorize geographic objects influences the concept used, for example: pond, lake, sea, and ocean. (3) The boundaries of several geographic objects are indeterminate, e.g. beach, mountain, and dune.

Maps cannot possibly show all geographic features found on the surface of the earth. The cartographer produces a map for a set of potential users. In response to their expected needs, the cartographer selects and highlights features which are deemed important for the intended class of potential map users and, correspondingly, omits other features. In mapmaking, these processes are subsumed under the term "cartographic generalization" [19]. In practice, maps are categorized often with respect to their potential use as street map, road map, ski route map etc..

4 Prediction of an appropriate Map with a Hilbert Space Model Within SCOP

This section applies SCOP to predict an answer for the question: "Which map is appropriate for a given context?", where the intended activities are used as contexts. The usage of the model is illustrated in Figure 2. A concept and a context serve as input parameters. The model calculates the collapsed state and returns it. In this collapsed state probability values for exemplars of the concept can be calculated.

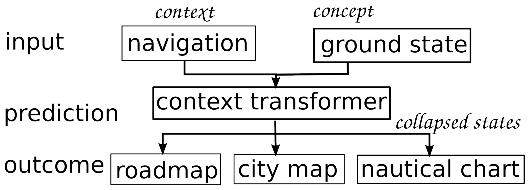

Fig. 2. Model for the prediction

The following example conversation may justify the example. A person states to another "Yesterday, I bought a map." What kind of map is meant remains undefined for the second person; the concept "map" is in ground state, where all maps have some non-zero probability to be meant. The first person continues: "I plan to go on a bicycling trip." Now the second person is influenced by the context, and the state of the concept "map" collapses into a bicycling map. The conversation may continue to indicate the region where the trip is planned, thus further restricting the map (beyond what is modelled here).

Using SCOP, a computational model for his conversation is possible. We implemented the relevant formulae [3] in the functional programming language Haskell [15], using the available matrix calculation packages (eventually using the standard implementations of GSL, BLAS or LAPACK).

To predict probability values, we define possible exemplars in Table 1 for the set Σ. The set Σ also includes the ground state $\widehat{p}$. For the term exemplar, SCOP also uses the term state of the concept.

Table 2 inherits properties for all elements giving set $\mathcal{L}$. In Table 3 the weights of the properties for each context are listed. These values are based on our own experience and appear realistic; the values are sufficient for a proof of concept, but are not the result of a representative experiment. Function ν uses this table.

The meaning of the concept "map" depends on the intended activity; the mapmaker creates the map for the intended purpose and typically labels it accordingly. We posit that users have, from experience, similar sub-categories for "map". Table 4 lists the intended activities the map should serve. These activities are the contexts included into set $\mathcal{M}$.

States of the concept, set Σ	**Kind of the map**
$\widehat{p}$	map
p1	roadmap
p2	hiking map
p3	city map
p4	nautical chart
p5	ski runway map
p6	bicycling map

Table 1. States of the concept map.

Properties for the concepts, set $\mathcal{L}$	**Layers of the map**
a1	road
a2	lake
a3	buildings
a4	mountains
a5	ski runs
a6	bicycling lanes
a7	hiking path
a8	contour lines

Table 2. Properties of the concept map.

The input parameters for SCOP are the frequency values included in Tables 5 and 3.

At the start of the conversation the concept map is in ground state $x_{\widehat{p}}$. In this state none of the possible states is preferred. This results in a probability for each state as found in the ground state, i.e. (1). The value 1800 consists of the sum of the states without any context $(342+252+..+252)$. The variable $|u\rangle$ indicates vectors of the Hilbert space. The sum identifies the selected vectors. In the ground state all vectors available in the set $\mathcal{M}$ are chosen.

$$|x_{\widehat{p}}\rangle = \sum_{u \in M} \frac{1}{\sqrt{1800}} |u\rangle \tag{1}$$

Weights for properties	**e1**	**e2**	**e3**	**e4**	**e5**	**e6**
a1 road	0.9	0.5	0.9	0.3	0.4	0.8
a2 lake	0.6	0.8	0.2	0.8	0.7	0.3
a3 buildings	0.7	0.5	0.99	0.5	0.5	0.01
a4 mountains	0.1	0.9	0.4	0.8	0.7	0.7
a5 ski runway	0.01	0.6	0.01	0.01	0.99	0.01
a6 bicycling lanes	0.4	0.6	0.5	0.1	0.01	0.99
a7 hiking path	0.01	0.99	0.4	0.01	0.4	0.6
a8 contour lines	0.1	0.99	0.1	0.6	0.7	0.5

Table 3. Weights of the properties by context

Contexts, set $\mathcal{M}$	**Activities for maps**
1	I choose a map
e1	I choose a map for navigation
e2	I choose a map for hiking
e3	I choose a map for sight seeing
e4	I choose a map for sailing
e5	I choose a map for skiing
e6	I choose a map for bicycling

Table 4. Activities used as contexts for maps

Exemplars	**e1**		**e2**		**e3**		**e4**		**e5**		**e6**		**1**	
	Freq.	States	Freq.	States	Freq.	States	Freq.	States	Freq.	States	Freq.	States	Freq.	States
roadmap	54	216	9	27	21	105	13	39	5	15	7	7	19	342
hiking map	0	0	77	231	2	10	0	0	2	6	0	0	14	252
city map	4	16	3	9	67	380	18	54	0	0	5	5	22	396
nautical chart	7	28	0	0	0	0	69	207	0	0	0	0	15	270
ski runway map	0	0	0	0	0	0	0	0	93	279	0	0	16	288
bicycle map	36	144	10	30	2	10	0	0	0	0	88	88	14	252
sum		**404**		**297**		**505**		**300**		**300**		**100**	**100**	**1800**

Table 5. Frequency values in percentages and Hilbert States

By influencing the ground state with the context e6 “I plan to go on a bicycling trip” the state collapses into state x_{p_6}, where 100 states are present.

$$|x_{p_6}\rangle = \frac{P_{e_6}|x_{\widehat{p}}\rangle}{\sqrt{\langle x_{\widehat{p}}|P_{e_6}|x_{\widehat{p}}\rangle}} = \sum_{u \in e6} \frac{1}{\sqrt{100}}|u\rangle \tag{2}$$

With the function μ the weight of each map type (p1.. p7) can be checked. This equation yields a value between zero and one. A value closer to one identifies a highly appropriate exemplar, a value close to zero an inappropriate one.

For example, to check if the nautical chart is an appropriate map in state x_{p_6}, the projector P_{e_4} for nautical charts is used.

$$\mu\left(p_4, e4, x_{p_6}\right) = \langle x_{p_6}|P_{e4}|x_{p_6}\rangle = 0 \tag{3}$$

For state x_{p_6}, the probability for nautical charts equals zero, whereas the probability for the bicycle map equals 0.88. This is calculated as:

$$\mu\left(p_6, e6, x_{p_6}\right) = \langle x_{p_6}|P_{e6}|x_{p_6}\rangle = 0.88 \tag{4}$$

In this state the weight of the properties can also be calculated with equation (5). To calculate the weight of a road in the state x_{p_6} the equation is:

$$\nu\left(x_{p_6}, a_1\right) = 0.8 \tag{5}$$

The property of a map to show roads has a weight of 0.8 in state x_{p_6} and is therefore an important property, in contrast to the property ski-run (0.01). This values indicates whether the map should include this layer or not. This example will therefore include roads and will exclude ski-slopes. The calculated relevance of a property could be used to produce maps on demand for particular activities (currently, maps on demand are produced when user select the layers explicitly, which is usually too demanding for non-technical users and introduces confusing jargon; who knows what “bathymetry” is and when it is used? - but it should be used on maps used for sailing and boating!)

If we take the above conversation to be between a potential map user and a map producing service, then SCOP could be included in the program and calculate the probability for given maps, given the known contexts. If a map type receives a clear preference, it can be produced for the user. If not, additional questions can be asked to obtain more context from the user. These contexts can be processed partially, as suggested by Weiser and Frank [26].

5 Prediction of an appropriate Map combining Cycling and Buying of a Map

In this section we combine the concept of “map” from the previous section with a concept of “buying things”; in this situation effects like the known “guppy-effect” can occur, and can be handled through the formalism of “entanglement” from quantum mechanics [7].

In both concepts “buying a bicycle map” occurs and connects the two. The frequencies from Table 6 and Table 5 declare the input values. Table 6 includes two contexts influencing the concept “I buy things”, modelled as Hilbert space $\mathcal{H}^{buy}$. Context f1 includes the context cycling. As a further step, context f2 appends the context map to f1. Context f2 will result in an entangled state with the second Hilbert space.

Context	**f1**		**f2**		**1**	
Exemplar	**I buy things for cycling**		**I buy a map for cycling**		**I buy things**	
	Freq	States	Freq	States	Freq	States
bread	1	4	0	0	30	900
milk	3	12	0	0	28	840
rain jacket	13	52	0	0	14	420
first aid kit	32	128	0	0	13	390
bicycle chain	21	84	0	0	8	240
cycle helmet	19	76	0	0	3	90
road map	4	16	13	13	1	30
bicycle map	7	28	87	87	3	90

Table 6. Frequency values in percentages and Hilbert States for $\mathcal{H}^{buy}$

The second Hilbert space $\mathcal{H}^{map}$ models the values from Table 5. The context e6 is selected for entanglement. Context e6 and f2 declare the same context shown by essentially equivalent statements. This is the foundation for entanglement, which brings different received informations into a single context. The following Equation (6) describes this mathematically.

$$e_6, f_2 \in M^{map,buy} \tag{6}$$

SCOP uses the tensor product to describe combined systems; the tensor product combines all possible combinations of the basic states [4]. The entanglement set $M^{map,buy}$ is defined including the states from f_2, e_6. To create this state the two concepts f_2 and e_6 are combined by the Cartesian product, where each element from f_2 is combined with each element from e_6. The entangled state is formulated by Equation (7).

$$|s\rangle = \sum_{u \in E^{map,buy}} \frac{1}{\sqrt{100}} |u\rangle \otimes |u\rangle \tag{7}$$

Projectors can be applied to this state, to predict an answer for: "If one buys a map for cycling, is this a bicycle map?". The answer predicted by the model is a probability value for the projected exemplar. For the exemplar bicycle map the following projector is used:

$$P_{F_2}^{map} \otimes 1^{buy} = \sum_{u \in F_2^{map}} |u\rangle\langle u| \otimes 1 \tag{8}$$

Applying this projector to the state s and reducing this state will result in the following equation:

$$|s'\rangle\langle s'|^{buy} = \sum_{u \in E_{f_2^{map}} \cap E_{f_2^{map}}} \frac{1}{100} |u\rangle\langle u| \tag{9}$$

By applying function μ to the reduced state, the probability for the bicycle map can be determined, which equals 0.88; much higher than the probability for roadmap, with a value of 0.07. The value is also higher than the probability in both independent contexts; entanglement connects the information gained in one and reinforces it in the other.

6 Conclusions and Future Research

Geographic concepts often exhibit prototype effects: the prototypical mountain to a Swiss person is not what a mountain in the Netherlands looks like. There are a great many similar effects - indeed it is hard to find a geographic concept which does not exhibit a prototype effect. The understanding of context effects is considered a major impediment for GIScience [18].

A computational model to deal with prototype effects is urgently needed. The selection of layers for maps is just one example of context effects: a prototypical map (say, a road map) serves many purposes, but by far not all. Berendt et al. described how to build maps depending on aspects of uses [8]. The experiments reported here show that with SCOP a computational solution to maps constructed from individual layers for particular purposes becomes possible.

SCOP is an appropriate model for formalizing concepts influenced by contexts, including the combination of concepts. Aerts [5] presented a further model using Fock spaces [9] to treat the disjunction of concepts.

The experiment reported here shows that the SCOP formalism of computing with contexts and combinations of contexts can be applied to geographic concepts. It promises (1) to help with the selection of maps for particular uses and (2) to contribute to the construction of maps on demand for a particular use without asking the user to construct the maps from individual layers.

Interesting and challenging research questions remain:

- Collect for several meaningful communities the data describing how they use the concept of map, following the example of Aerts and Gabora [3].
- Extend the example from before to include the selection of maps depending on the intended location of an activity. This increases the number of ground states for the concept "map", namely by regions (e.g. road map for Italy, France...)
- Users do not desire specific map types, they request maps with certain informations which are relevant for their planned activities. Instead of using SCOP to identify the map type suitable for an intended activity, one could identify the map layers which contain relevant information for the planned activity directly, and produce customized maps for different activities. Selecting the layers to be shown seems possible with SCOP, leaving the issues of graphical interactions between the map layers; the customized map must not only include the desired layers, but present them in a form which allows reading the presented information!
- Apply the SCOP computational model to other geographic concepts, e.g. town, village, mountain, forest, and observe how this affects statistical data collected across communities with different conceptualizations of e.g. forest. What is the correct answer to the question of the total forest area of Europe, if one exist considering the differences in the concept "forest" ?

For inclusion in a practical system, the SCOP formalism could be further developed into an incremental algorithm; in particular, give computational solutions to adding one additional ground state or an additional context (the first seems difficult, the second trivial).

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